

Learning the Index vs. Indexing the Learned Models? A Tutorial on Learned Multi-dimensional Indexes

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Outline of the Tutorial

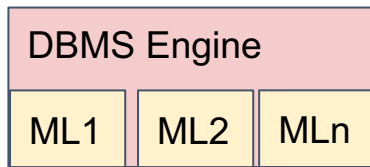
- Introduction and Taxonomy
- Indexing the Learned Models vs. Learning the Indexes
- Static vs. Dynamic Learned Indexes
- Fixed vs. Dynamic Data Layout
- Learned One-Dimensional Indexes
- Learned Multidimensional Indexes
- Open Problems for Future Research

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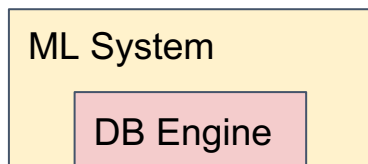
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Introduction: ML and DB

- Machine Learning (ML) has been successful in many application domains
- Two recent trends of research in the area of Database Systems (DB):



ML for DB

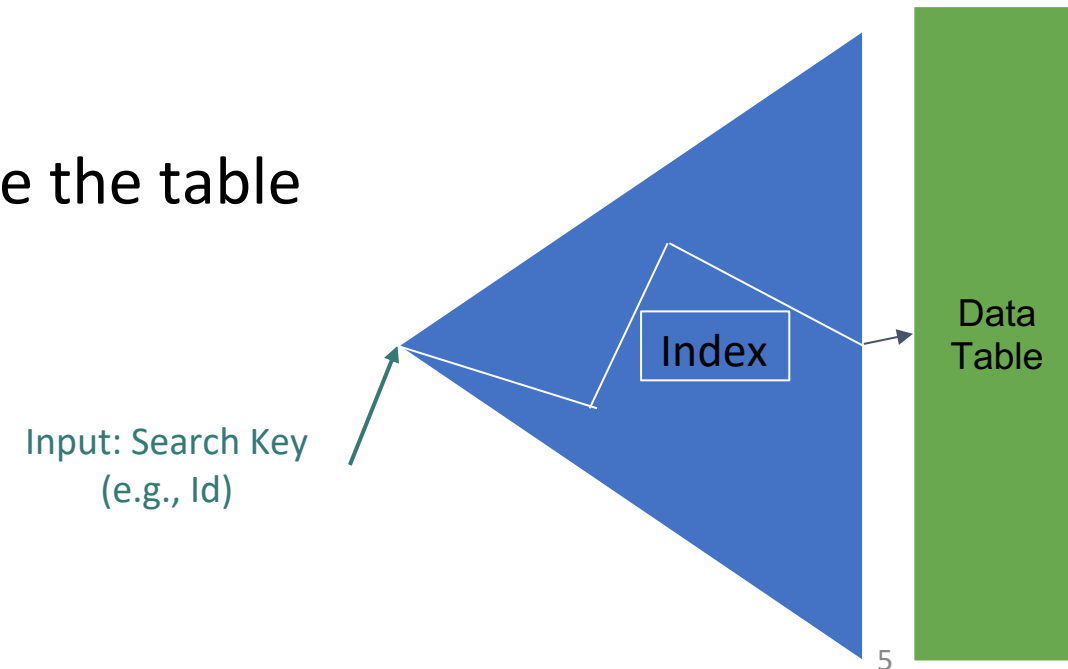


DB for ML

- *Machine Learning for Database Systems (ML for DB)*
 - Replace core components of a Database System (e.g., query optimizer, Indexes, DB administration) with Machine Learning techniques
 - Achieve better performance
 - Less space requirement
- *Database Systems for Machine Learning (DB for ML)*
 - Extend database system techniques to support efficient ML workloads

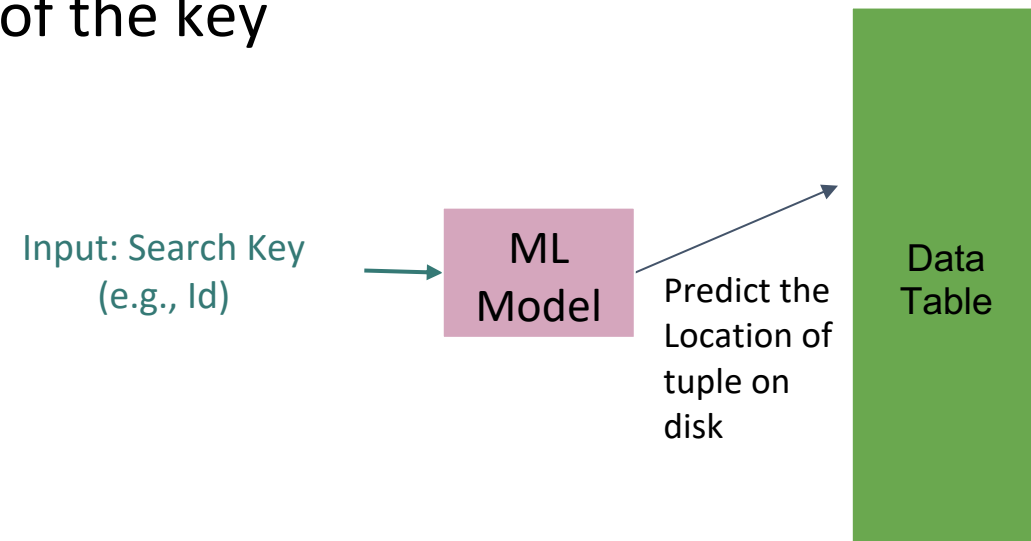
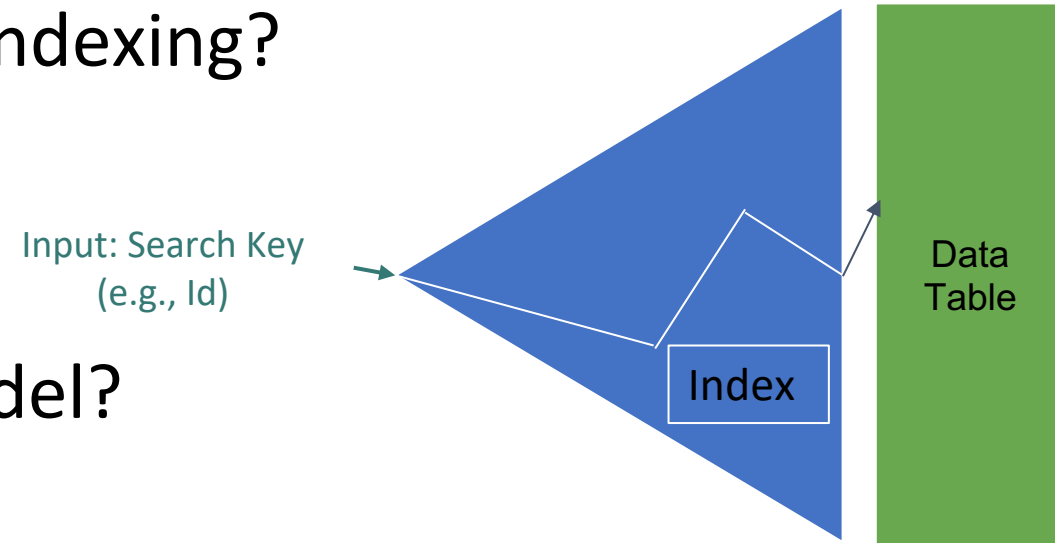
Introduction: Database Indexing

- Database Index: Provide efficient access to data
- Popular index structure is: B⁺-tree
- Given search key, B⁺-tree identifies the storage location of the tuple that contains the search key
- Can view the B⁺-tree as a function:
 - B⁺-tree(key) → the order of key's tuple inside the table



Introduction: Learned Index

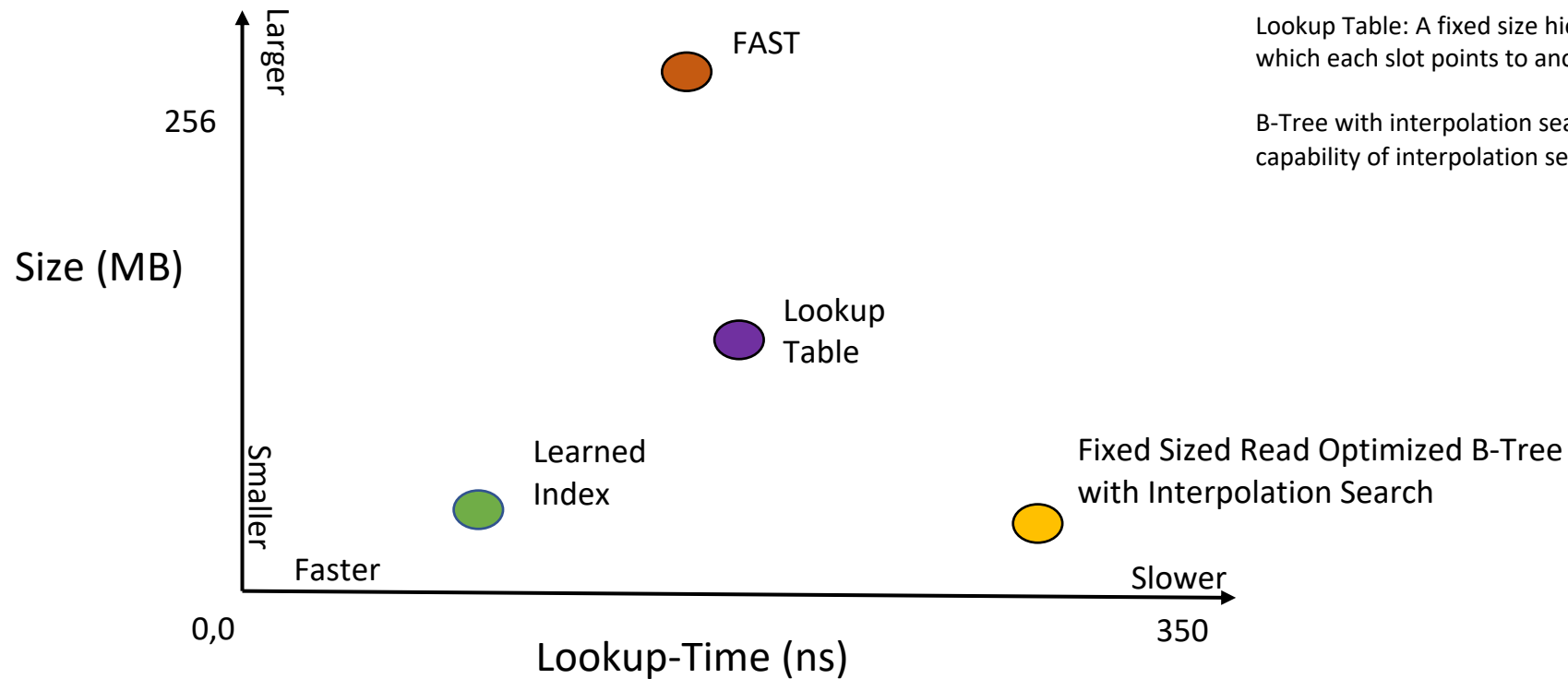
- Can one use ML techniques to guide data indexing?
- Can we learn the function:
 - B^+ -tree(key) \rightarrow Location of tuple in table?
- Can we replace the B^+ -tree with an ML model?
 - “Index as a model”
 - $ML_Model(key) \rightarrow$ predicts the storage location of the key
 - Searching executes potentially in $O(1)$ time
 - \rightarrow “Learned Index”



Introduction: Initial Results

Initial performance results (approximate) of a learned index

Promising → Faster lookup time and smaller storage



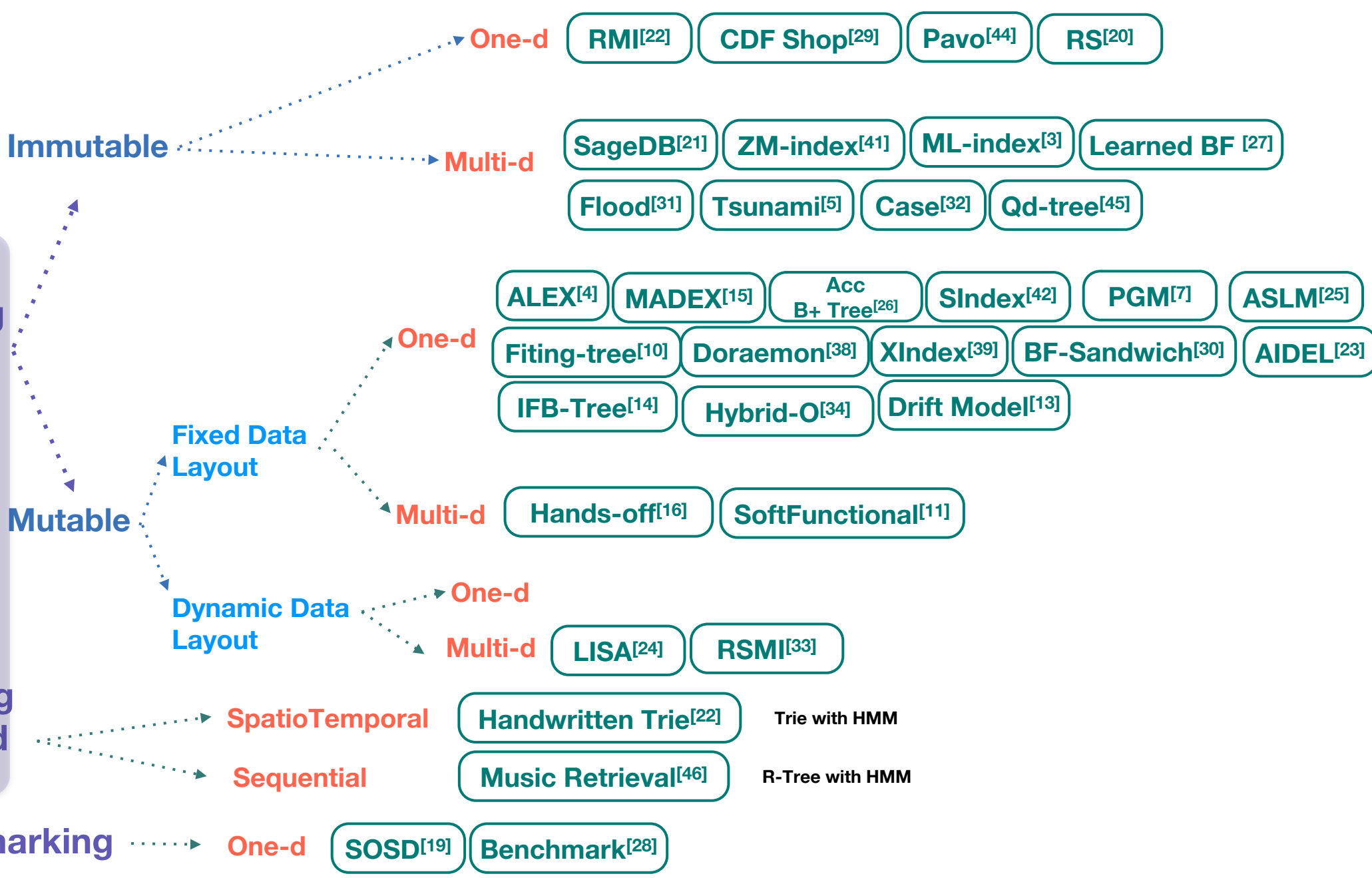
FAST: A highly SIMD optimized data structure.

Lookup Table: A fixed size hierarchical structure (e.g., 64 slots for which each slot points to another 64 slots, etc.).

B-Tree with interpolation search: A fixed-height B-Tree with the capability of interpolation search. It is optimized for read operations.

Taxonomy of Learned Indexes

Learning the Index
LEARNED INDEX
Indexing Learned Models



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Taxonomy of Learned Indexes

Dimension 1: Indexing the Learned Models vs. Learning the Index

Machine Learning and data indexing interact in two possible ways:

1

Indexing the Learned Models

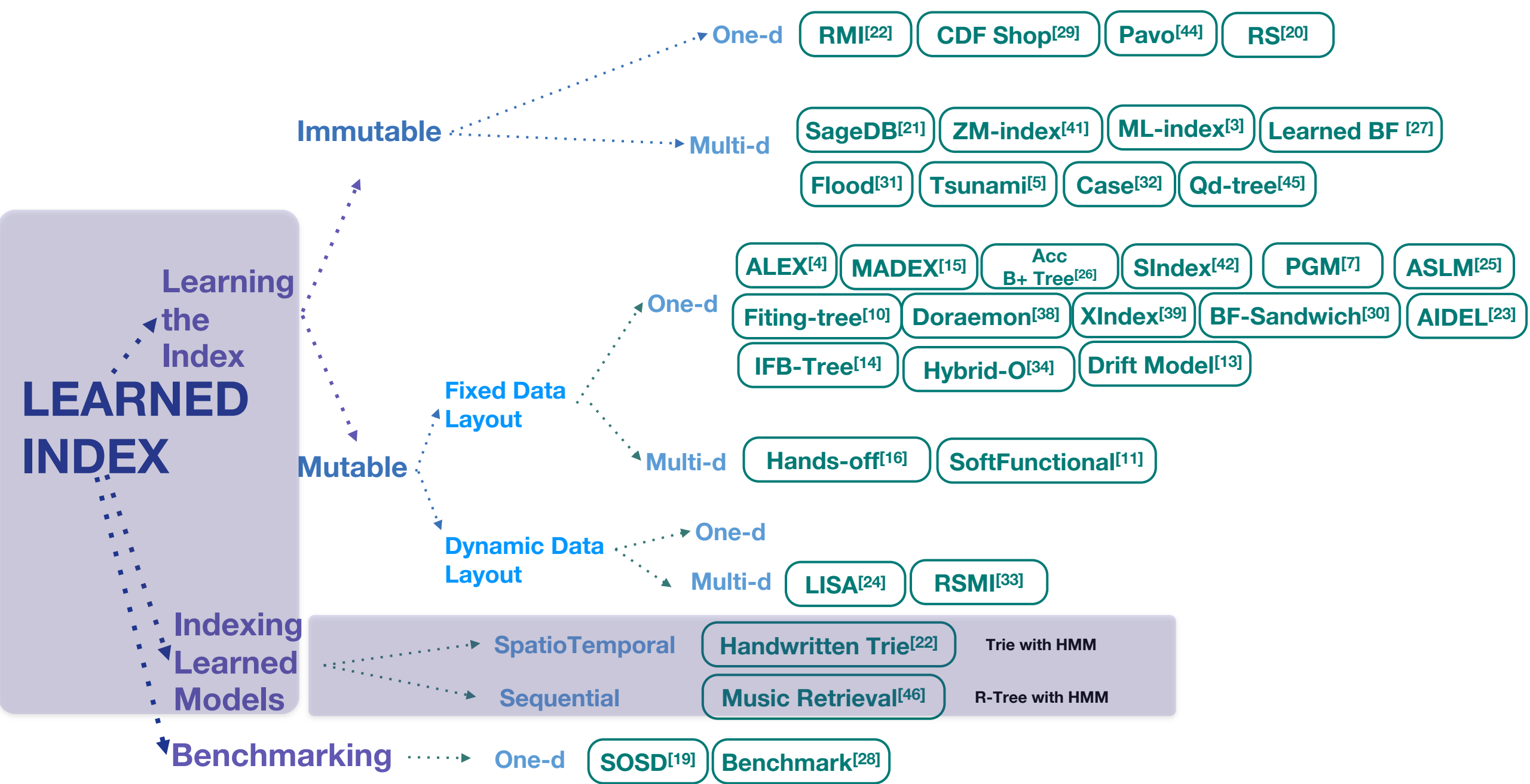
- Given a collection of ML models, e.g., object recognition models (Cats, Dogs, Trains, etc.), and an input object, say o
- Identify the class of o (cat vs. dog, etc.)
- Instead of executing all models and identifying which has the highest matching score
- Can we index the learned models to speed up the matching process?

Learning the Index

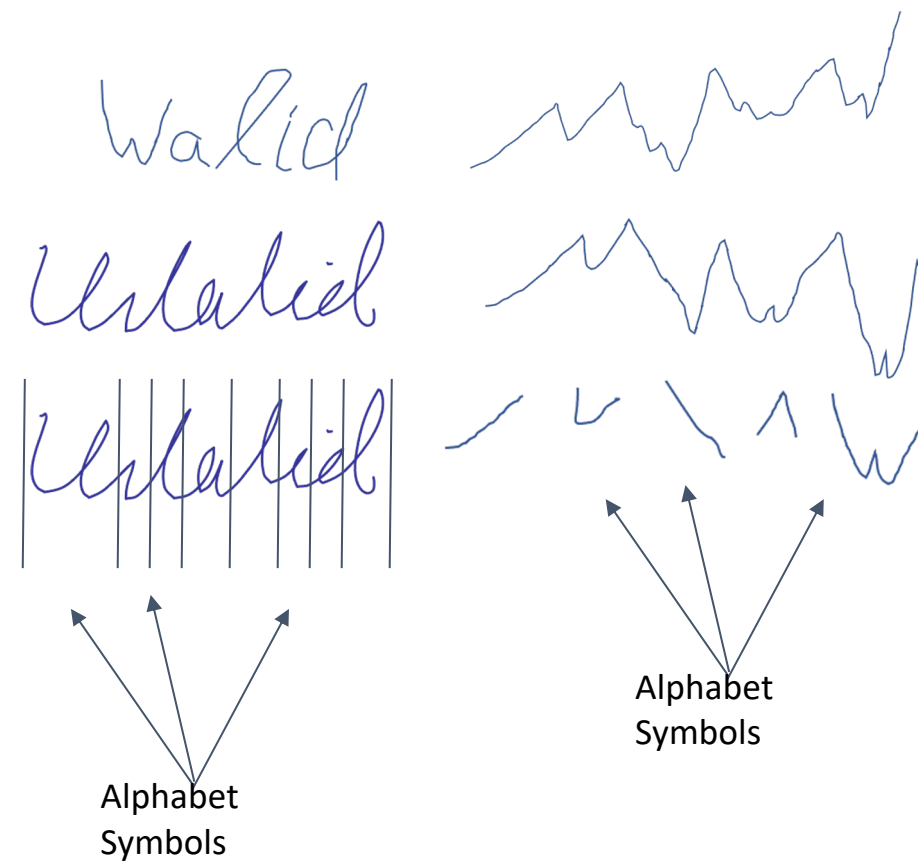
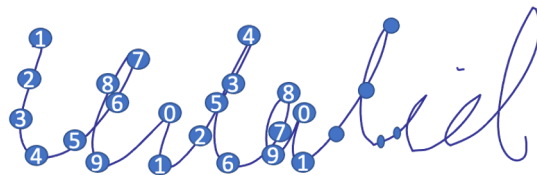
2

- Given a key value, say k , and an ordered array of key values
- Build an ML-based model that helps predict the location of k in the ordered array

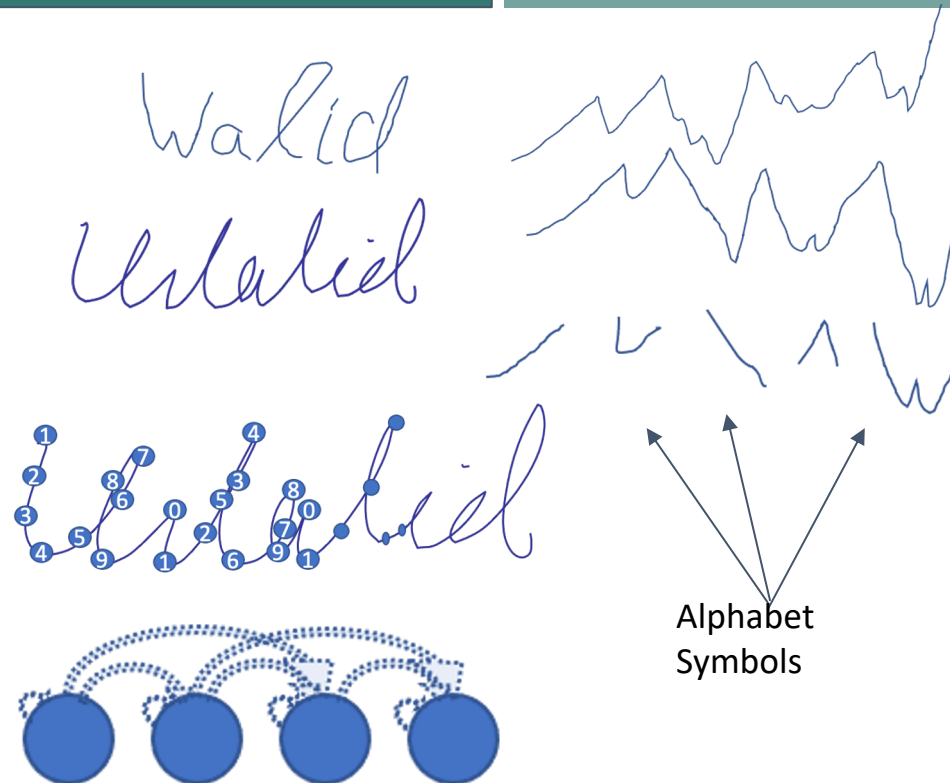
Taxonomy of Learned Indexes



- Collection of spatiotemporal sequences, e.g., heart pulse rates, stock market trends over time, handwritten drawing on a tablet, object movement trajectory
 - Index shown in the context of handwritten text
- Divide each spatiotemporal sequence into basic alphabet symbols
- Because of the variability, there is a need for training to recognize similar, but not exactly the same, patterns
- Model each alphabet symbol in the spatiotemporal sequence using *local spatiotemporal features* along the trajectory of the sequence
 - Time
 - Velocity
 - Direction
 - Acceleration
 - Aspect ratio, . . .



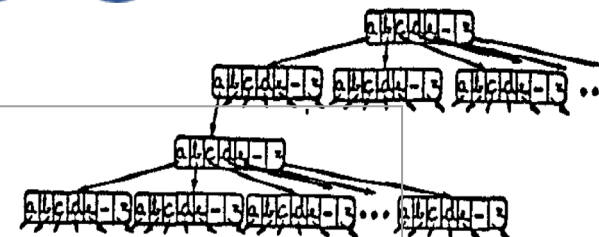
- Left-to-right Hidden Markov Models are suitable for representing spatio-temporal sequences
- Instead of building a Hidden Markov Model for each entire sequence, we build an HMM for each alphabet symbol in the spatiotemporal sequence
- Need to segment each spatio-temporal sequence into alphabet symbols
- Train the left-to-right Hidden Markov Model using multiple samples of the alphabet symbols
- Construct a trie structure over the learned alphabet symbols



Alphabet Symbols

Core Idea

- Indexing the learned Hidden Markov Models
- Trie Structure over learned alphabet symbols



Hidden Markov Model

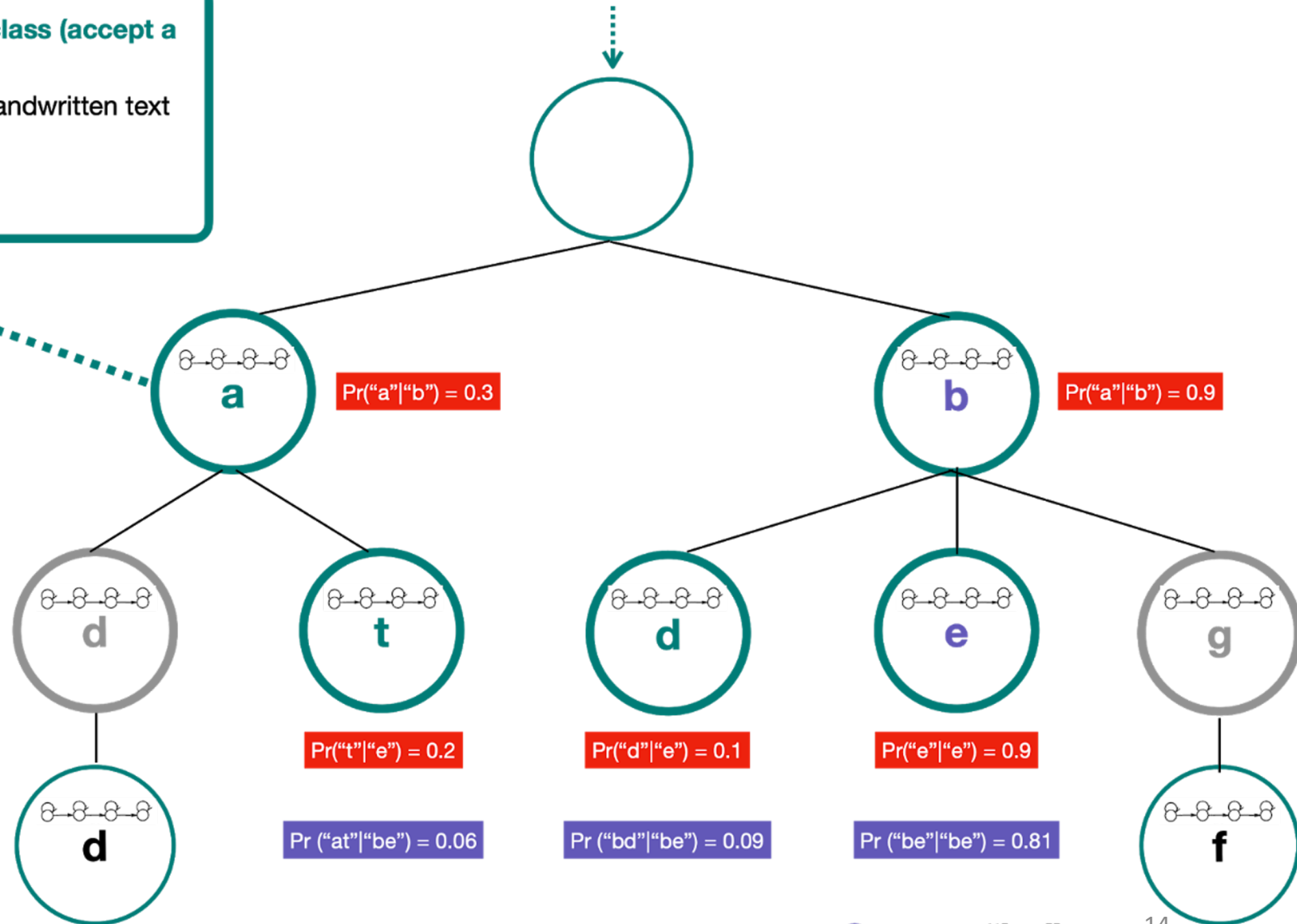
Each HMM is constructed to represent a pictogram class (accept a specific pictogram with high probability)

- **Left-to-right HMM:** Useful for modeling cursive handwritten text
- **Input:** Pictogram
- **Output:** Matching Probability

Input: "be"

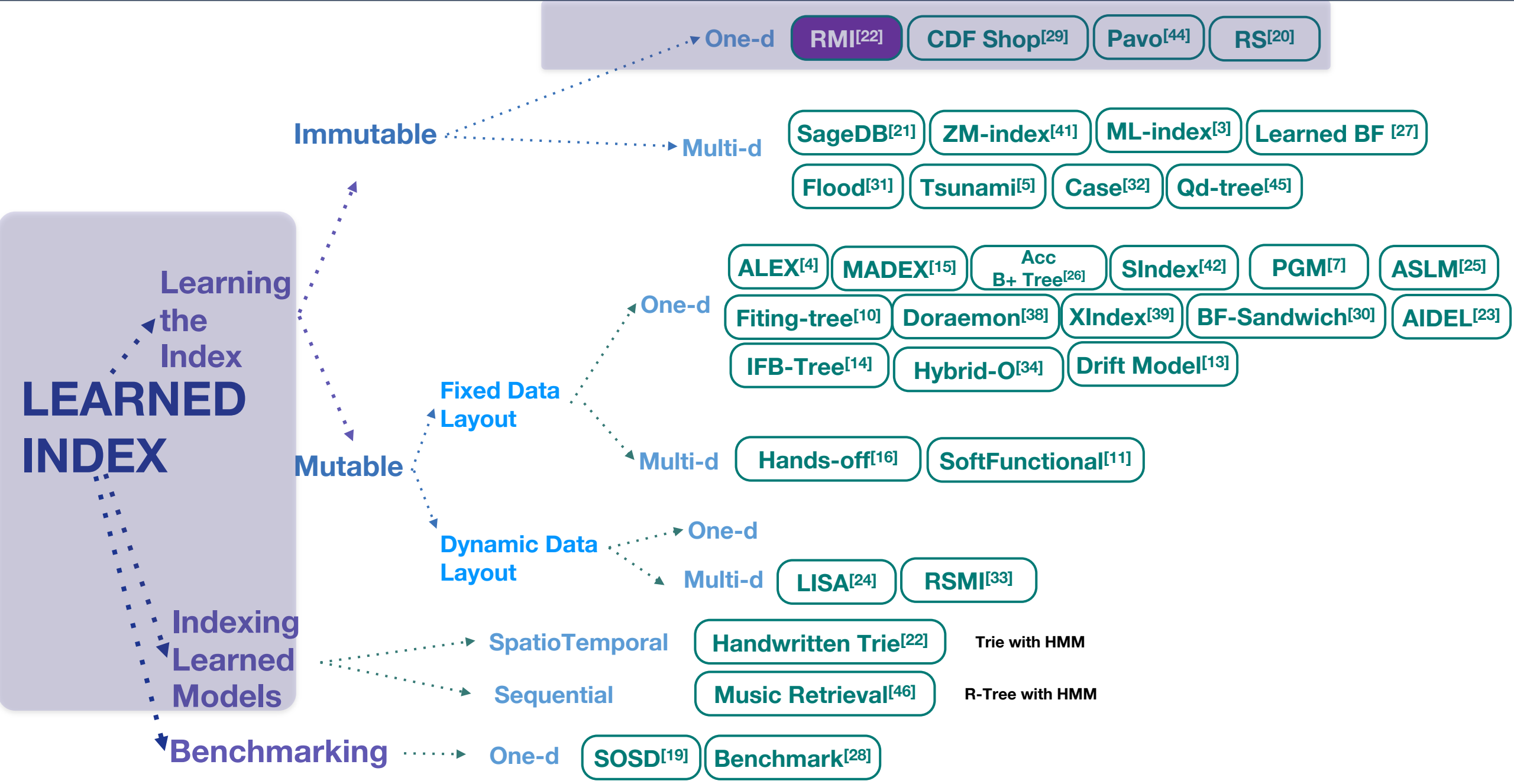
Traverse Nodes (HMM Models)
Choose Nodes with highest probability

choose combination of nodes with highest probability

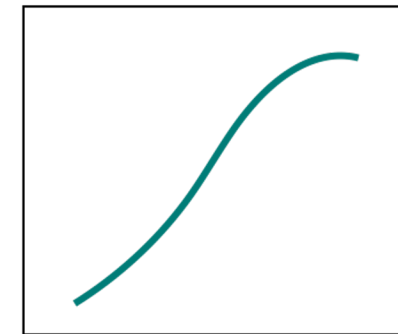


- The Handwritten Trie: Indexing Electronic Ink is one of the earlier works to index the models.
- Another early work about indexing the models using R-Tree-like structure in the context of indexing HMMs for music retrieval can be found in [ISMIR'02]

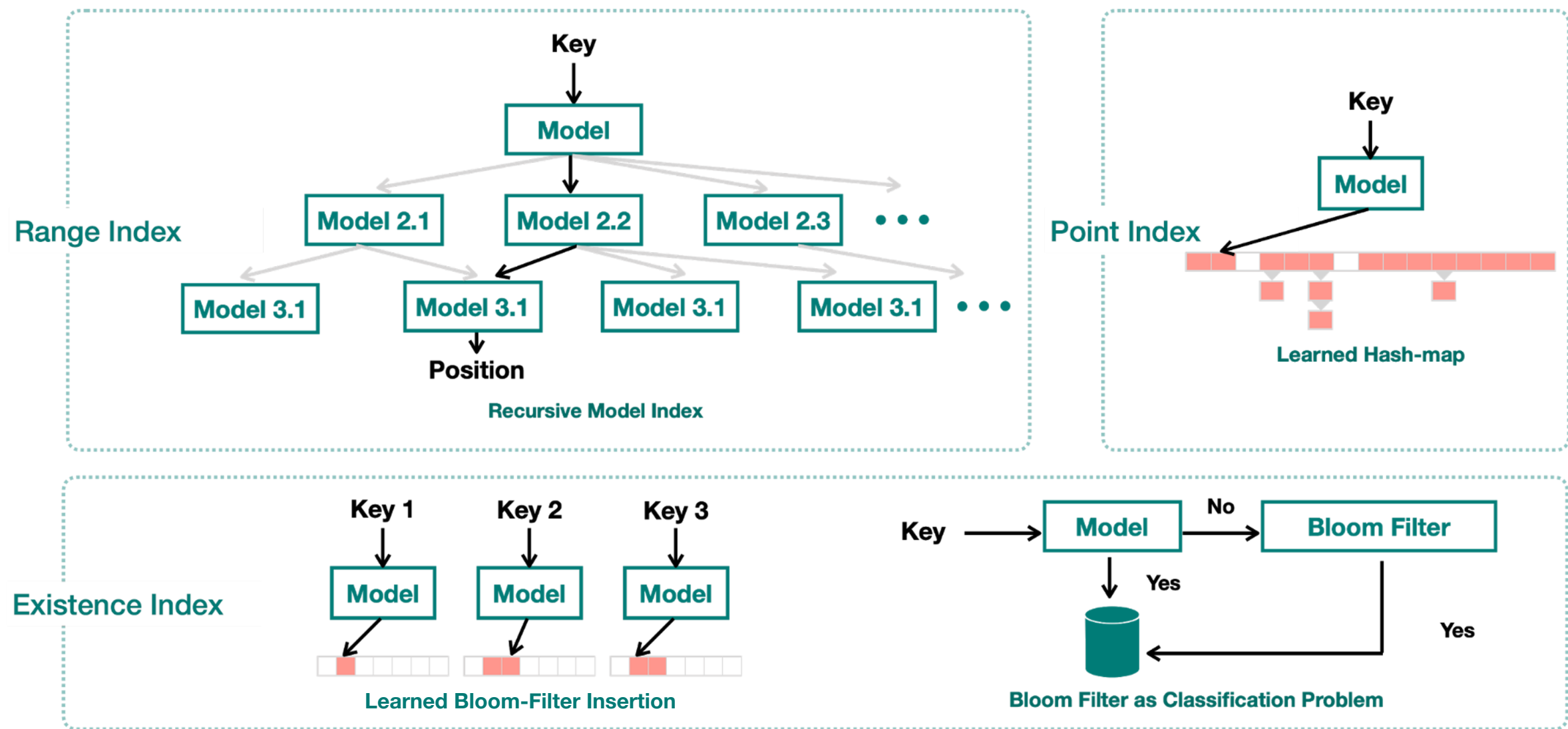
Taxonomy of Learned Indexes



- Introduced the idea that “Indexes are models”
- Replace traditional database indexes by learned models
- Approximate the Cumulative Distribution Function (CDF) of the underlying (sorted) data
- Proposed Recursive Model Index (RMI), a multi-stage ML model
- Combine simpler ML models
 - The first stage model will make an initial prediction of the CDF for a specific key
 - The next stage models will be selected to refine this initial prediction
- Proposed Learned Index Structures: Range Index, Point Index, and Existence Index



CDF



- Limitations:
 - Focus on in-memory read-only workloads
 - The structure of RMI is static
 - Does not support updates (e.g., insertion, deletion)
- Many follow-up works extend on this paper

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Dimension 2: Immutable vs. Mutable Learned Indexes

Given a Learned Index, can we support updates?

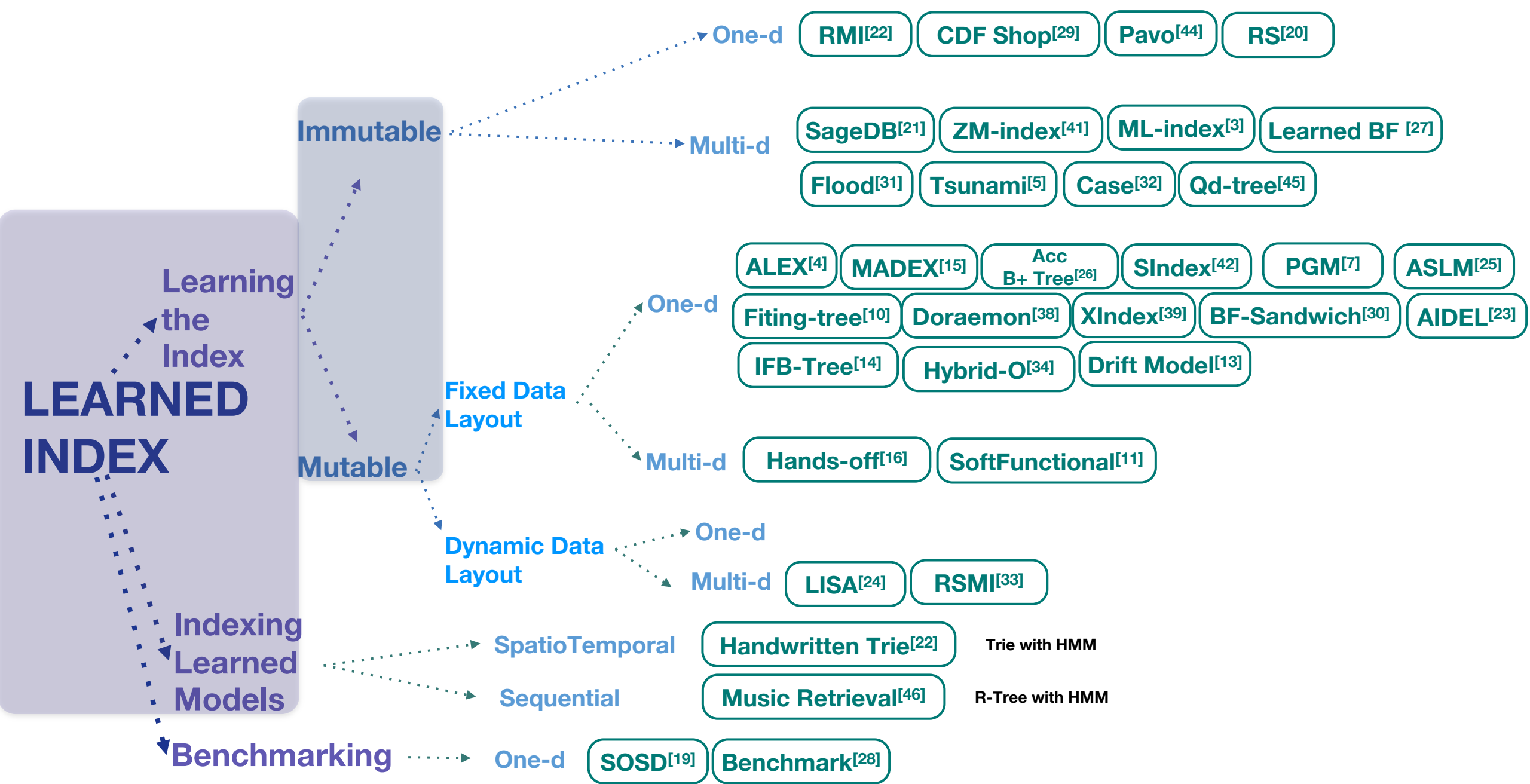
Why are updates hard?

- The learned index takes significant time to train
- New data will require retraining because it changes the order of the data

Classify learned indexes based on the ability to support updates:

- 1 *Immutable Learned Index:*
 - Does not support inserts, updates, or deletes
- 2 *Mutable Learned Index:*
 - Supports inserts, updates or deletes

Taxonomy of Learned Indexes



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Dimension 3: Fixed vs. Dynamic Data Layout

There are two types of data layouts for learned indexes:

1

Fixed Data Layout:

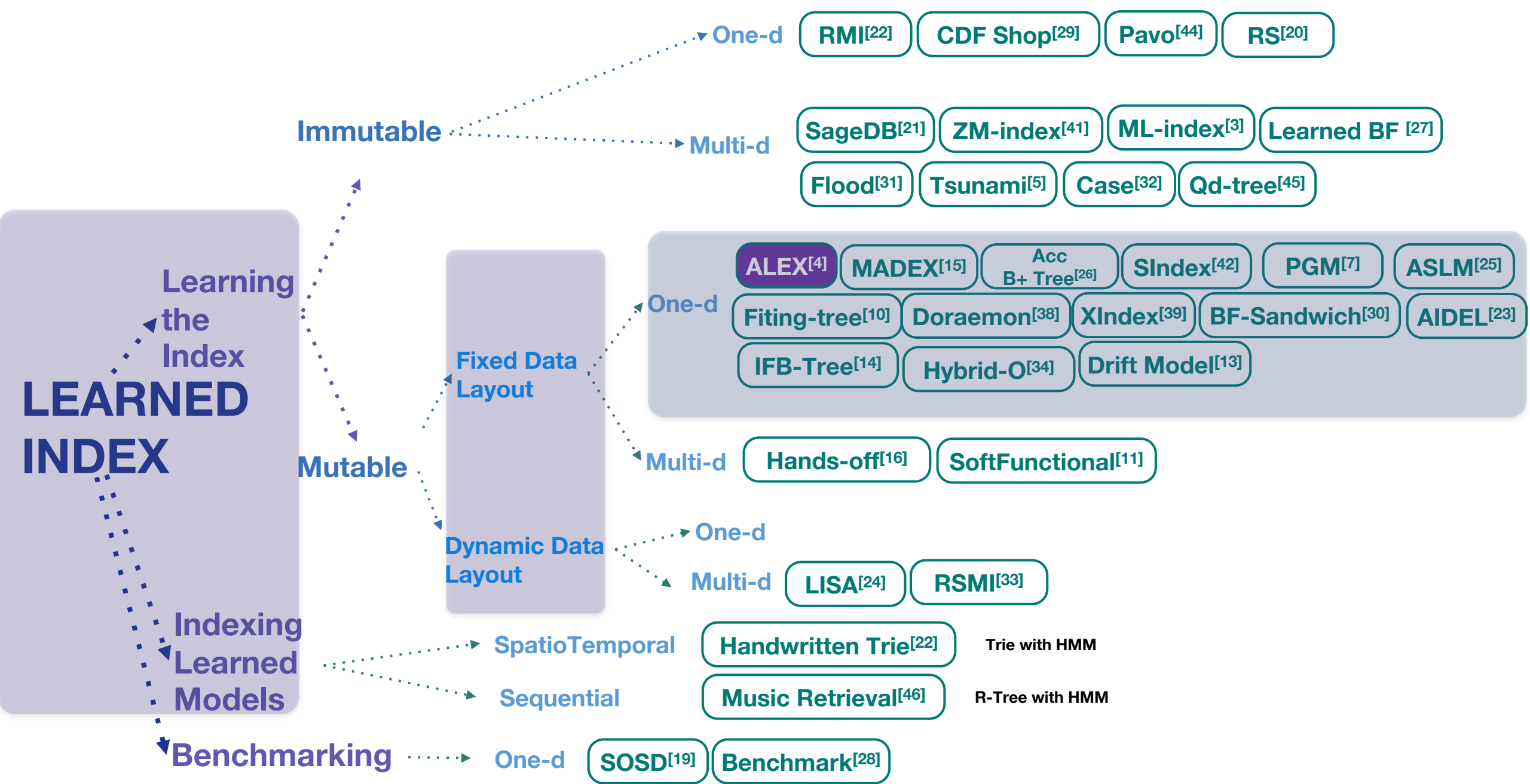
- The layout of the data and the structure of the index is fixed before building the learned index

2

Dynamic Data Layout:

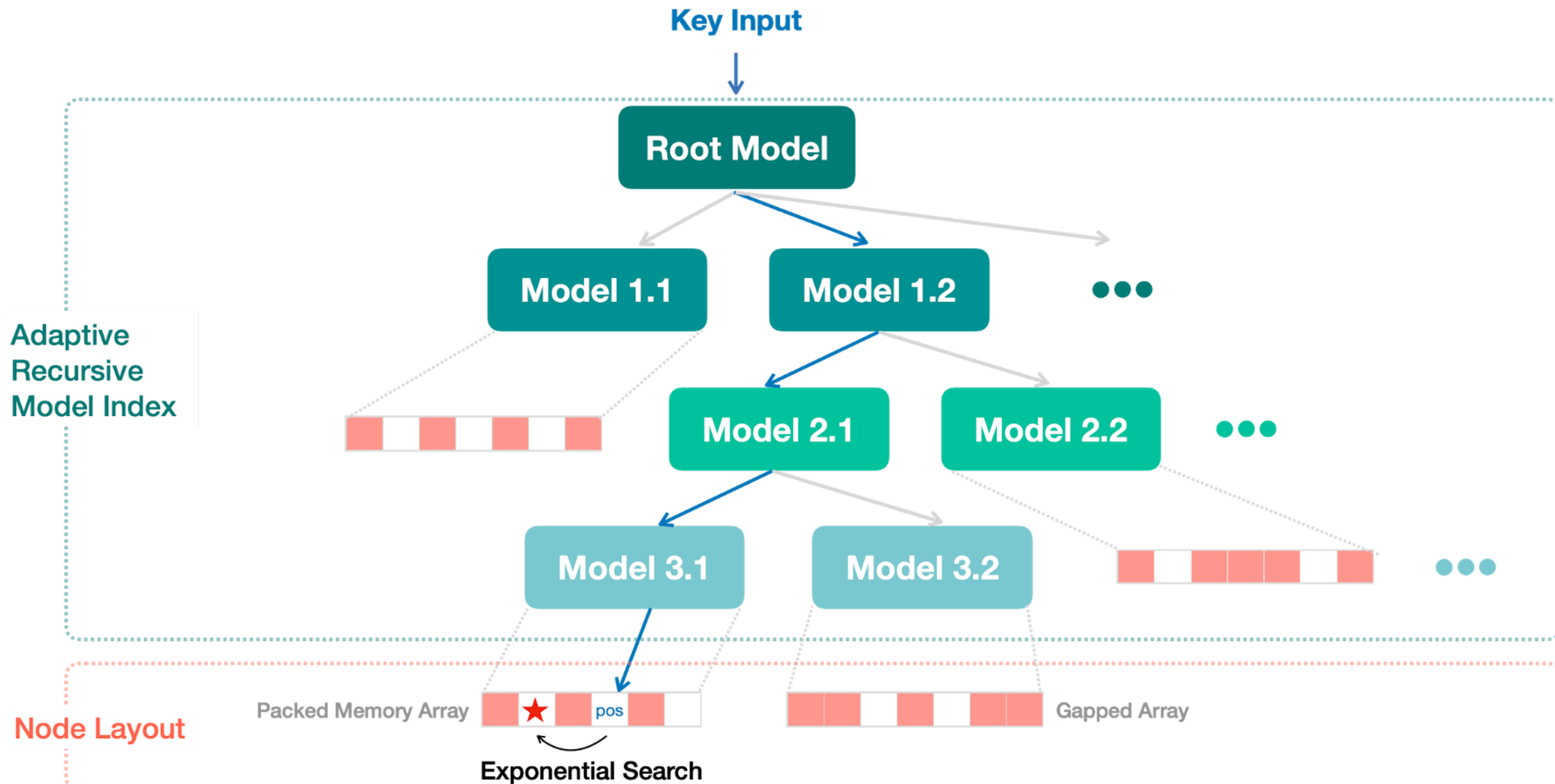
- The layout of the data is arranged and can be modified by the ML models while building and updating the learned index

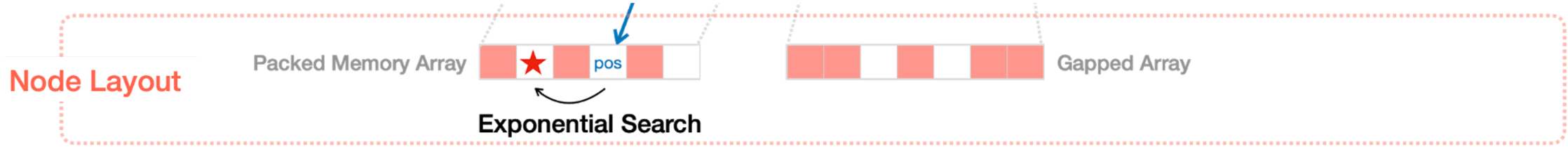
Taxonomy of Learned Indexes



Target	Learned Index (Stable & Dynamic Workload)
Objectives	Implement dynamic, updatable learned index to handle dynamic workload
Core Idea	<ul style="list-style-type: none">• Adaptive RMI as Model Hierarchy• Linear Regression Model as Node• Gapped Array or Packed Memory Array as Node Layout

[4] Jialin Ding, Umar Farooq Minhas, Hantian Zhang, Yinan Li, Chi Wang, Badrish Chandramouli, Johannes Gehrke, Donald Kossmann, and David Lomet. SIGMOD 2020. ALEX: An Updatable Adaptive Learned Index. arXiv preprint arXiv:1905.08898(2019).

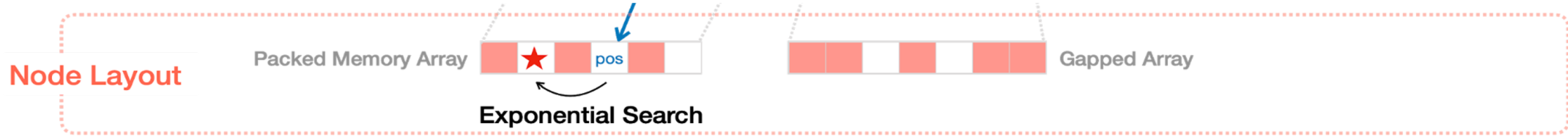




1 Gapped Array (GA)

	1	2	3	5		7	8		11
--	---	---	---	---	--	---	---	--	----

- Gapped Array(GA):
 - A sorted array where there are empty spots (gaps) among elements
 - Density of a Gapped Array is bounded by a preset limit.
 - During insertion:
 - Case-1: Predicted position is a gap, and maintains the sorted order
 - The key will be inserted.
 - Case-2: inserting the key in the predicted position cannot maintain sorted order
 - Exponential search will be used to find the correct position.
 - Case-3: predicted position is already occupied
 - Elements will be shifted by one position toward the nearest gap.

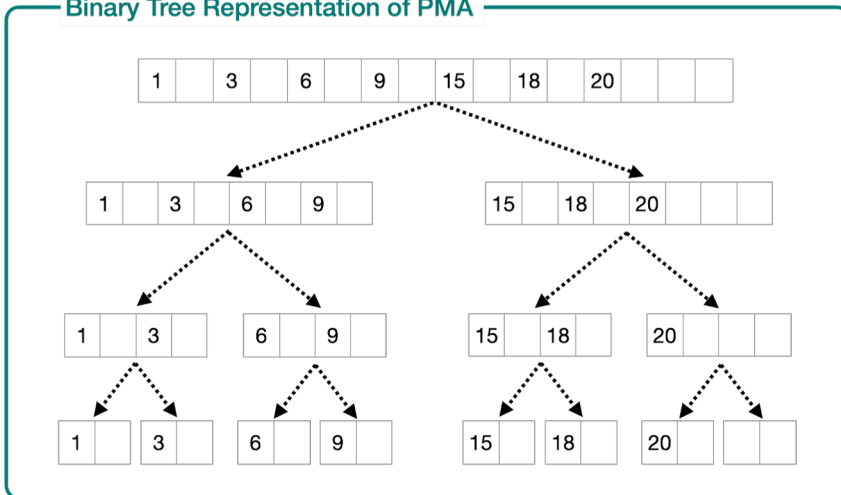


2

Packed Memory Array (PMA)



Binary Tree Representation of PMA



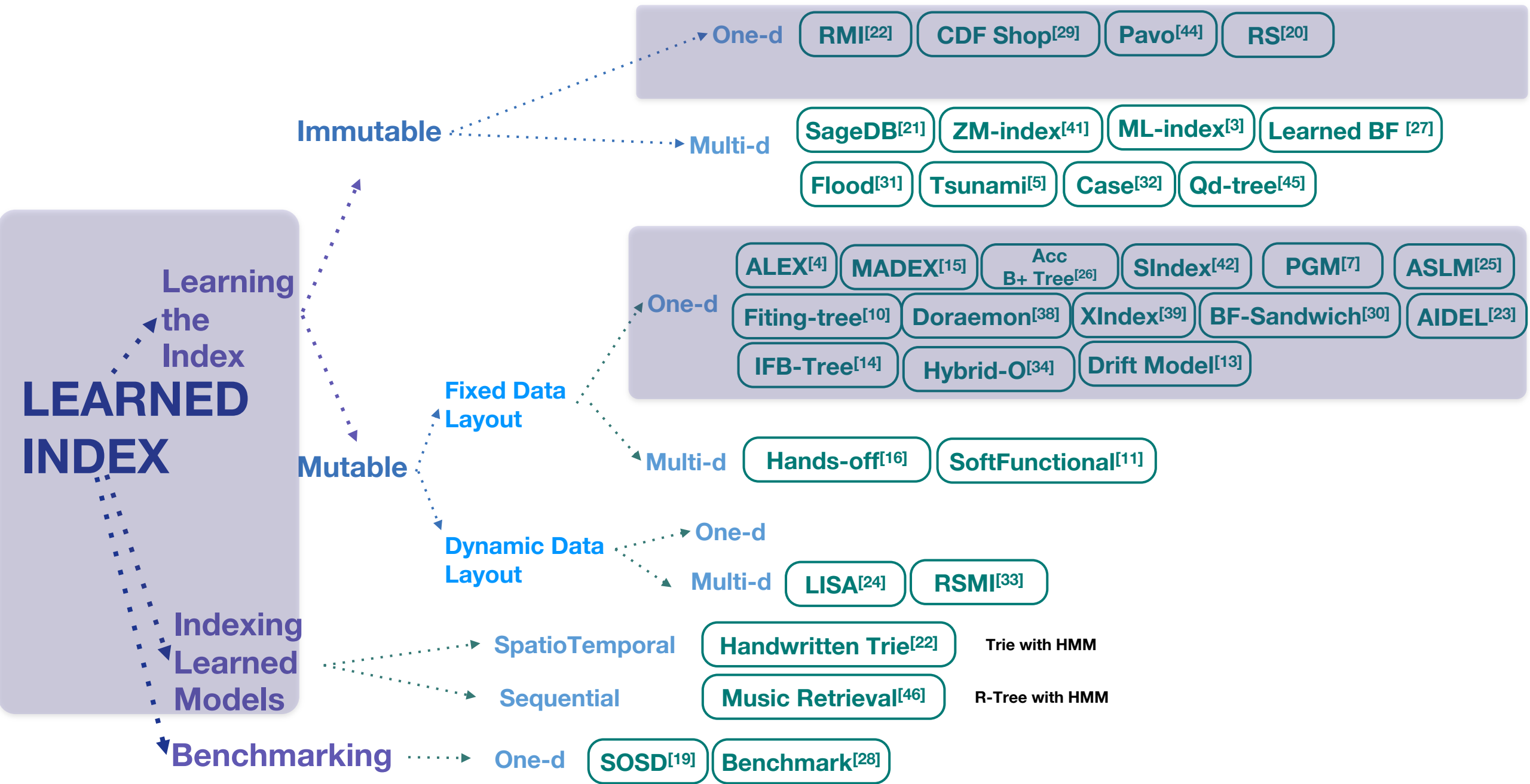
- Packed Memory Array (PMA):
 - A sorted array with size of power of 2 where gaps are uniformly spaced between elements.
 - It has an implicit binary tree representation
 - PMA places density bounds on each node of this implicit binary tree
 - The closer a node is to the root, the lower its density bound.
 - During insertion, If a segment's density bounds are violated:
 - Find some local region of the array
 - Uniformly redistribute all elements within this region
- GA vs. PMA:
 - PMA negates the benefits of model-based inserts.
 - During rebalancing, PMA spreads the keys in the local region over more space.
 - Worse search performance because the keys are moved further away from their predicted location.

- Limitations:
 - Adversarial workload when data is skewed
 - ALEX does not support duplicate keys of secondary indexes
 - Requires concurrency control to handle updates with concurrent lookups
 - How to check sorted order during insertion in gapped array (linear search?)

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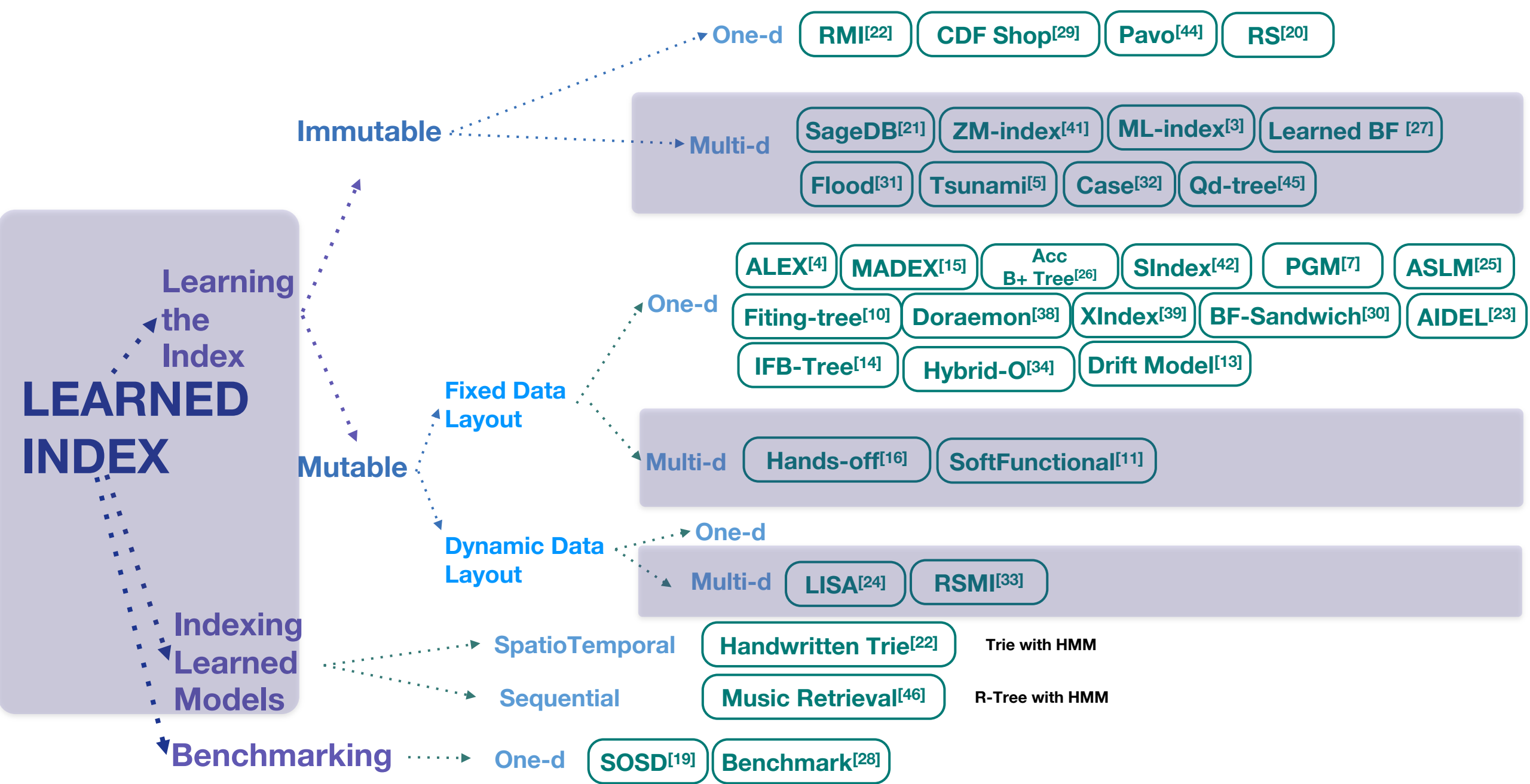
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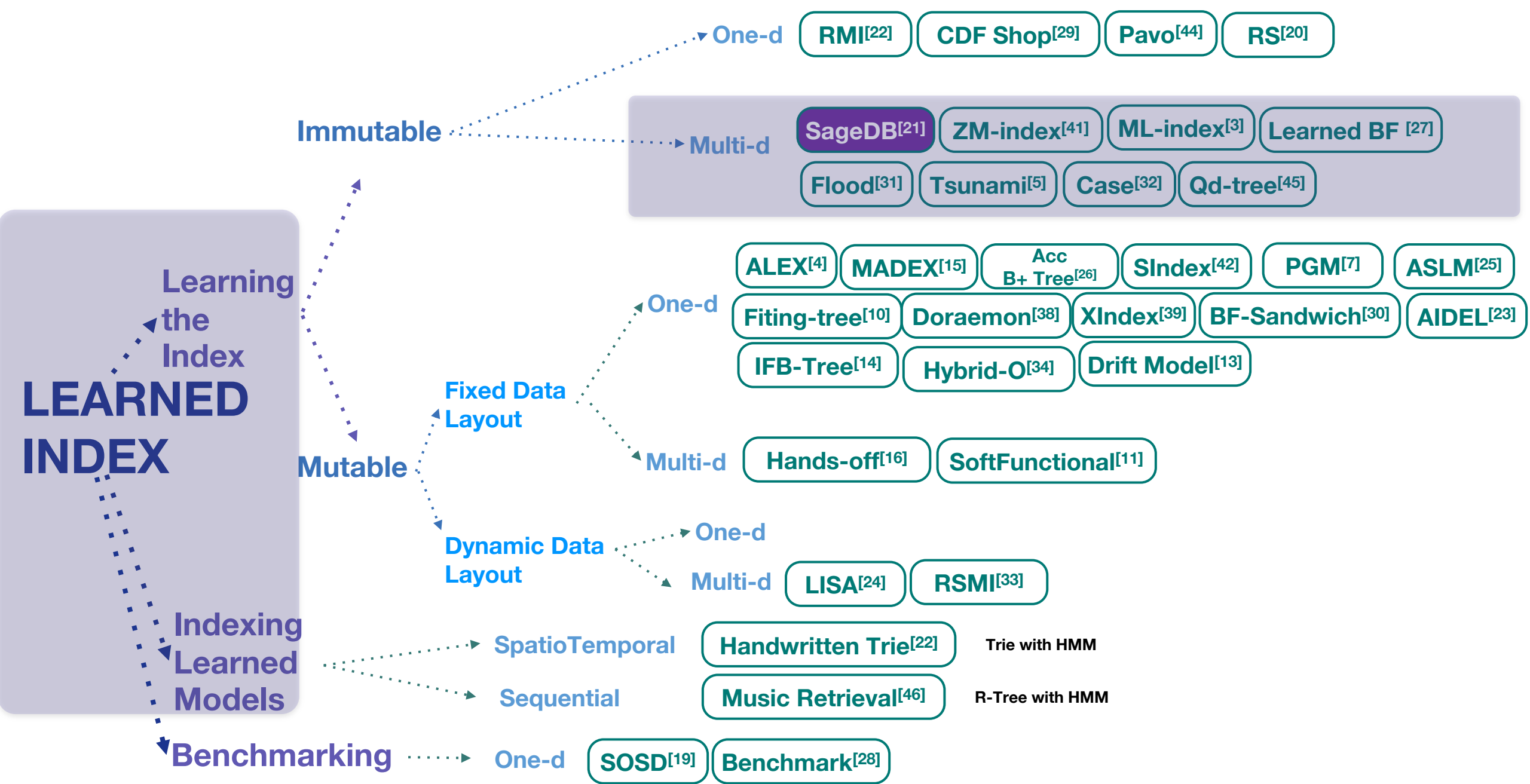
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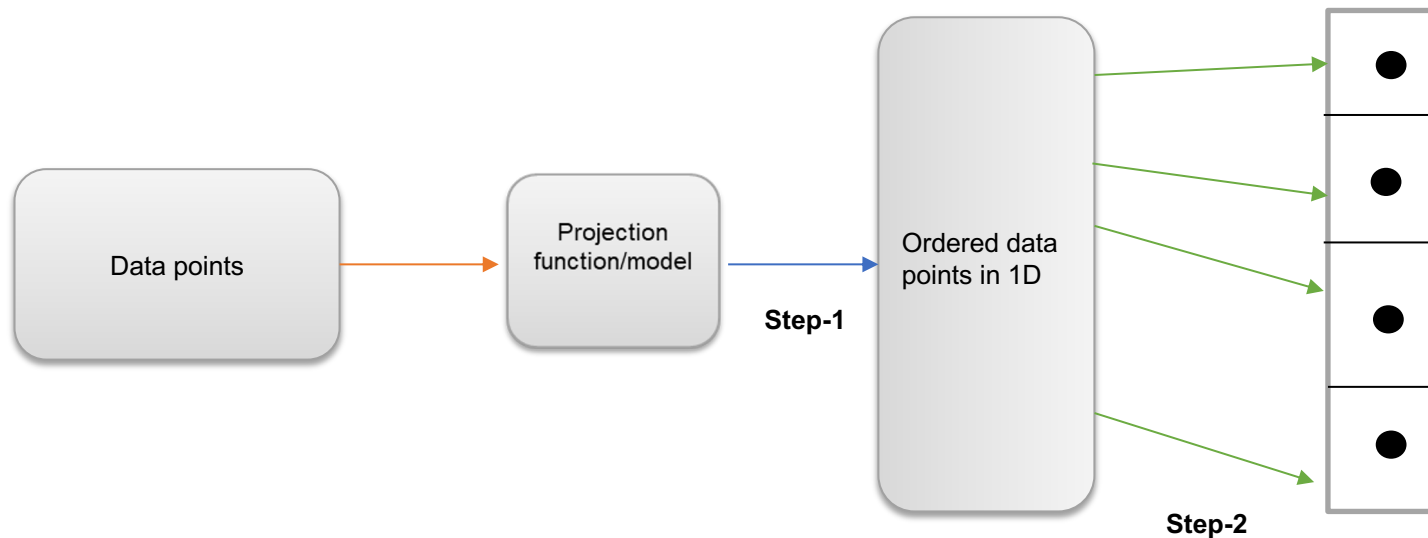
- One of the earliest distribution-aware spatial indexes can be found in:
 - [47] Babu, G. Phanendra. "Self-organizing neural networks for spatial data." Pattern Recognition Letters 18, no. 2 (1997): 133-142.
- Can ML models replace and act in place of a multi-dimensional index?
 - Yes, ML models has the potential to act in place of a multi-dimensional index, e.g., R-Tree.

- *Sorting/ordering of multi-dimensional data:*
 - No obvious sort order for multi-dimensional data
- *Error correction mechanism in case of misprediction:*
 - Difficult to define an error correction mechanism in case of mispredictions
- *Choice of the ML model:*
 - Which ML models to choose?
- *Layout of the data:*
 - How to store the data?
 - Affect range query time and model accuracy

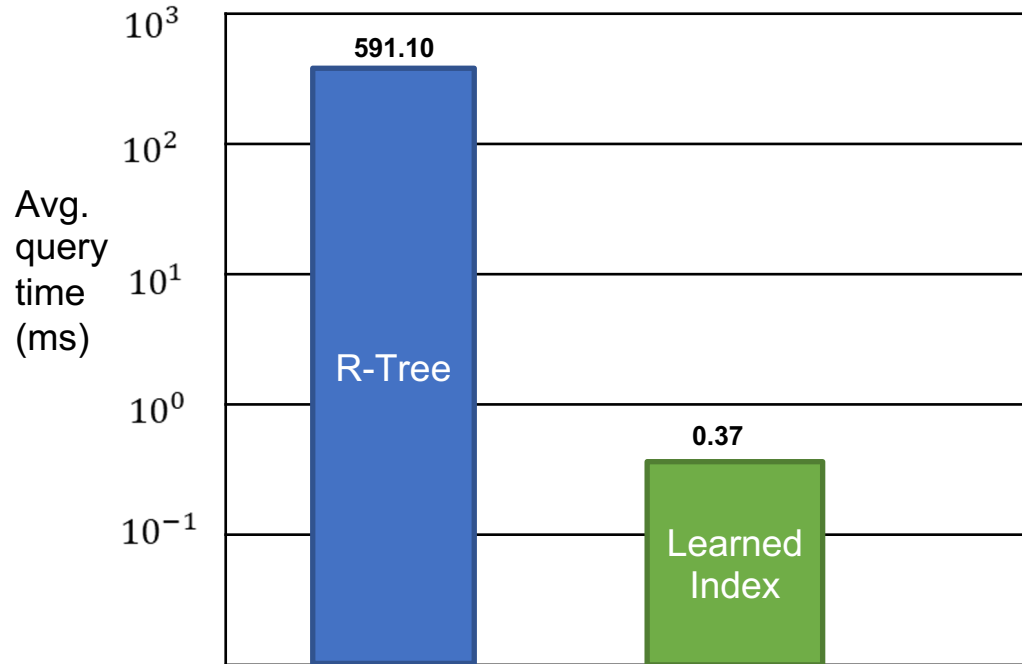
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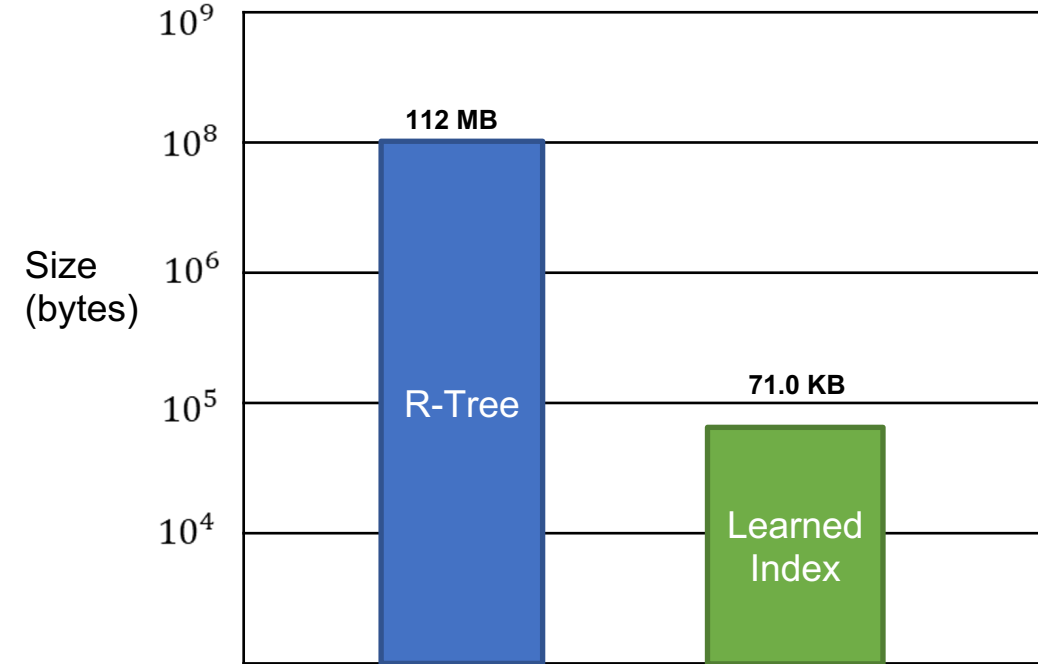
- Proposal:
 - *Step-1*: Project the multi-dimensional data points into one-dimensional space
 - Successively sorting and partitioning points along a sequence of dimensions into equally-sized cells
 - Produces a layout that is efficient to compute and learnable
 - Comparing with Z-order which is difficult to learn
 - *Step-2*: Uses a trained CDF model (e.g., RMI) to predict the physical location of the point



- Initial Result:
 - R-Tree vs. Learned Multi-dimensional Index on TPC-H data



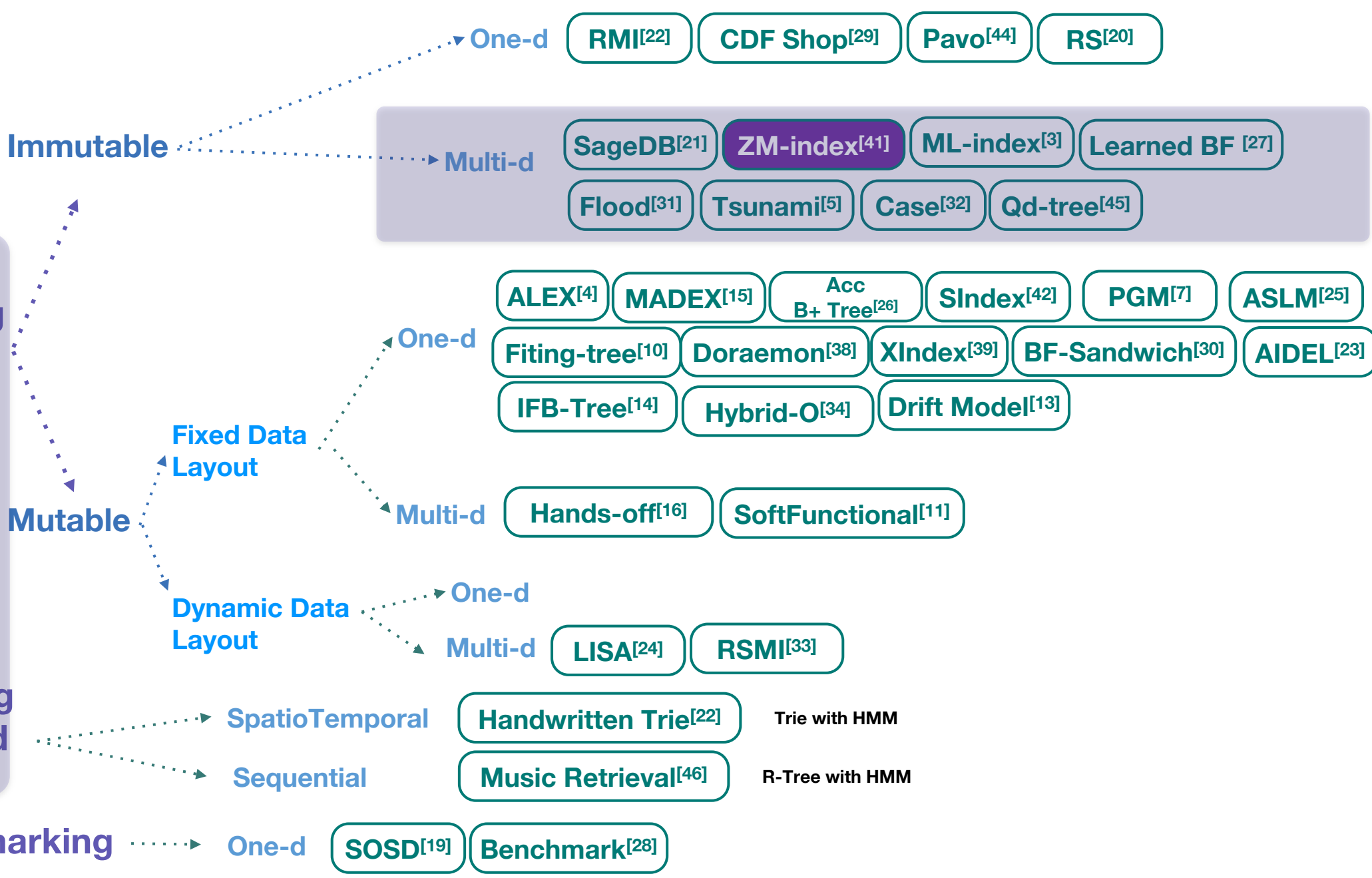
Result on average query time

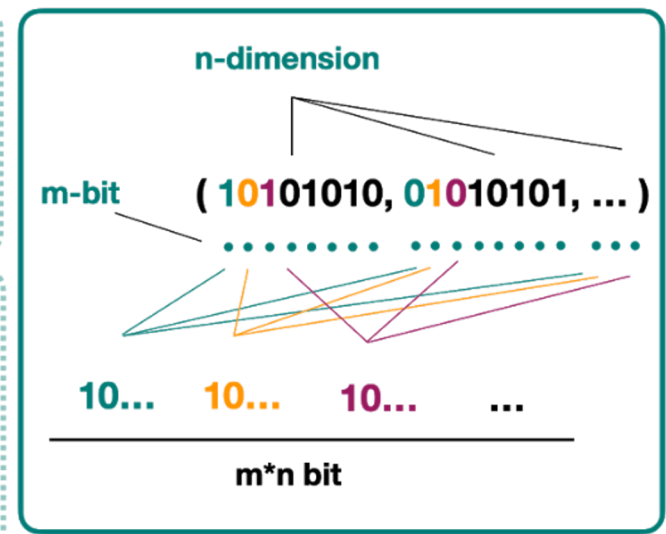
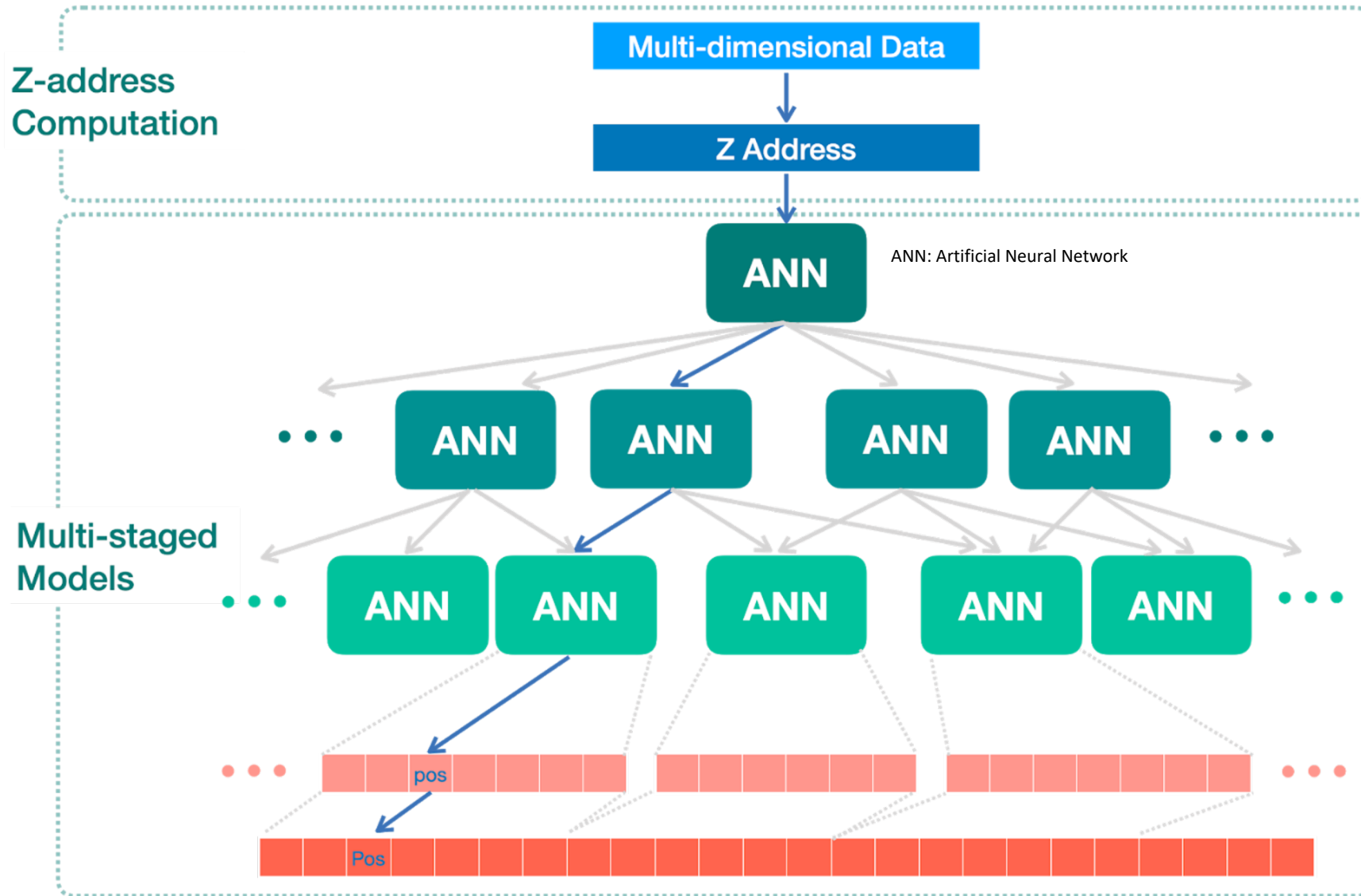


Result on index size

Taxonomy of Learned Indexes

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Z-address computation of n-d vector

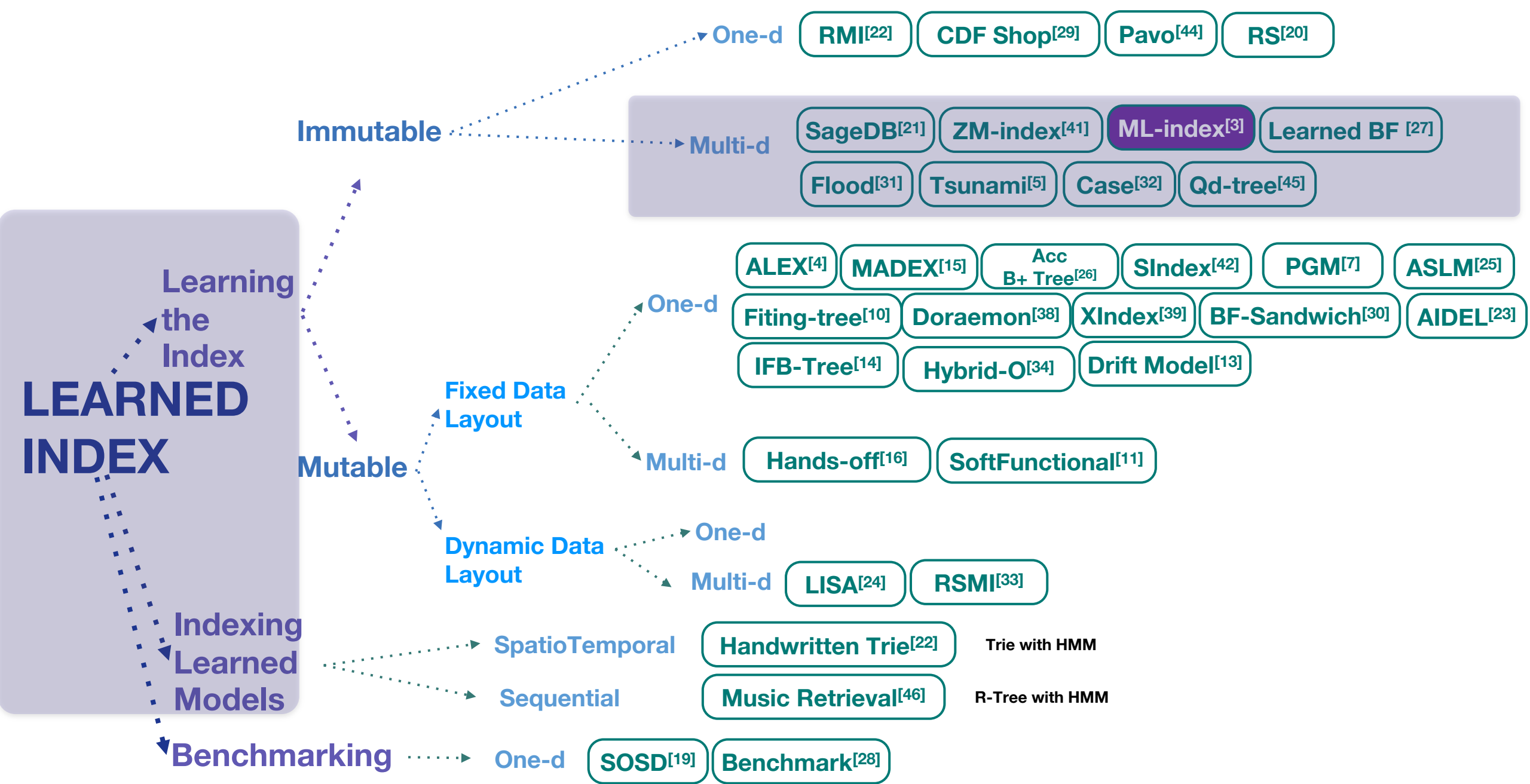
Core Idea

ZM-Index:

- Spatial Query Processing: Point and range queries
- Uses the Z-order to map the multi-dimensional values to the one-dimensional space
- Uses a multi-staged model (e.g., RMI) for learning

[41]Haixin Wang, Xiaoyi Fu, Jianliang Xu, and Hua Lu. 2019. Learned Index for Spatial Queries. In 2019 20th IEEE International Conference on Mobile Data Management (MDM). IEEE, 569–574.

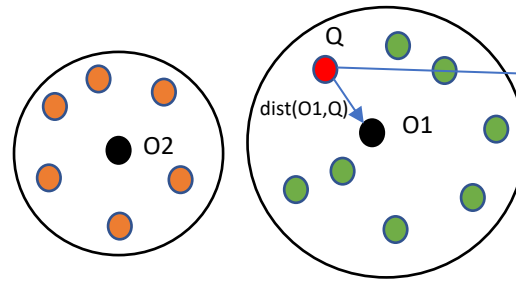
Taxonomy of Learned Indexes



Core Idea

ML-Index:

- Z/Morton order cannot be easily learned by ML models.
- Multi-dimensional data should be sorted in an order that can be easily learned.
- Partition and transform the data into one-dimensional values based on distribution-aware reference points.
- Combines the scaled ordering with ML models



Key = $\text{dist}(O1, Q) + \text{offset1}$

1. Find the closest reference point O_i and calculate the scaled value.

ML Model

2. Model (key) \rightarrow predicted position.



3. Local search

Position - error

Predicted position

Position + error

Query Processing (Point)

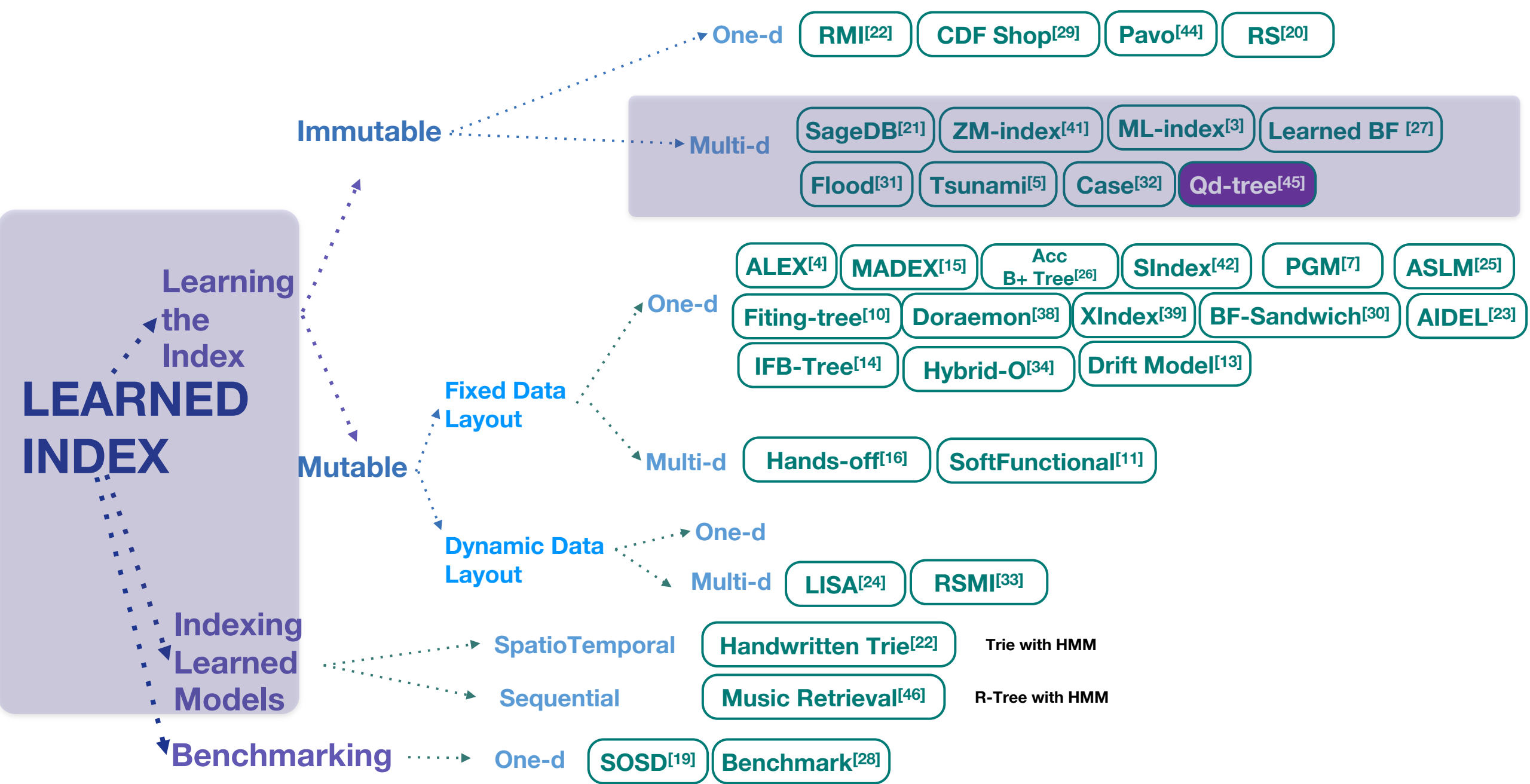
Efficient Scaling

Offset Method:

- m reference points O_i are chosen each can be thought as a centroid of the data partition P_i .
- The closest points of O_i are used to build the partition P_i .
- The minimal distance of a point to the reference points is d_i
- Scaled value = $\text{offset}_i + \text{dist}(O_i, d_i)$
- For reference points O_1, O_2, \dots, O_m and their partitions P_1, P_2, \dots, P_m ,
- r : The maximal distance from O_j to the points in partition P_j

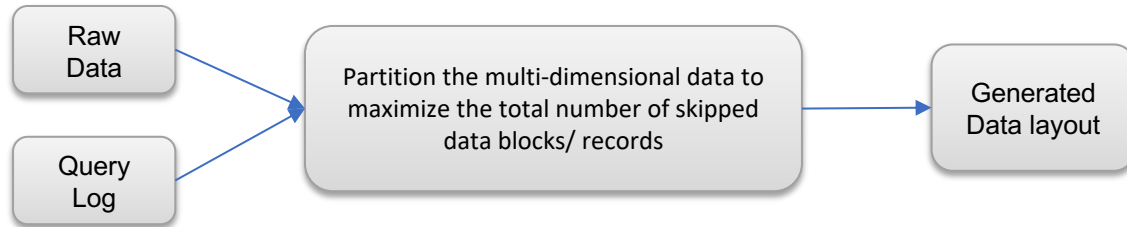
$$\text{offset}_i = \sum_{j < i} r(j)$$

Taxonomy of Learned Indexes



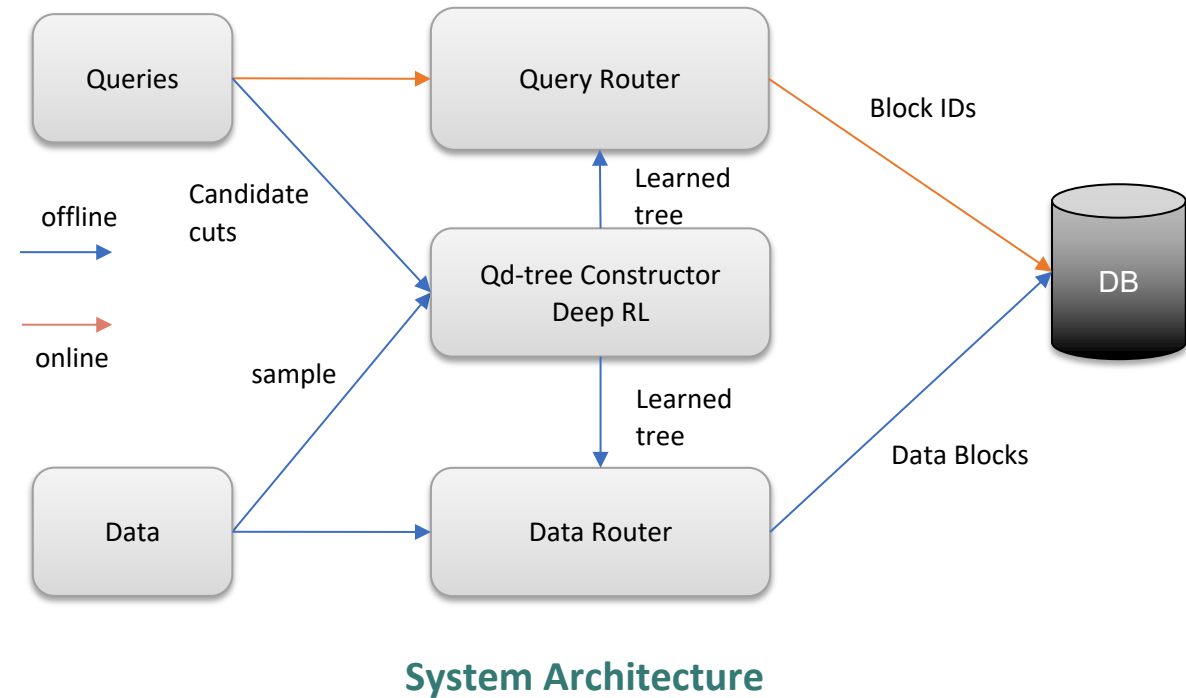
Motivation

- For disk-based systems, an important performance metric is:
 - The number of data blocks accessed by a query.
- Problem Statement:



Core Idea

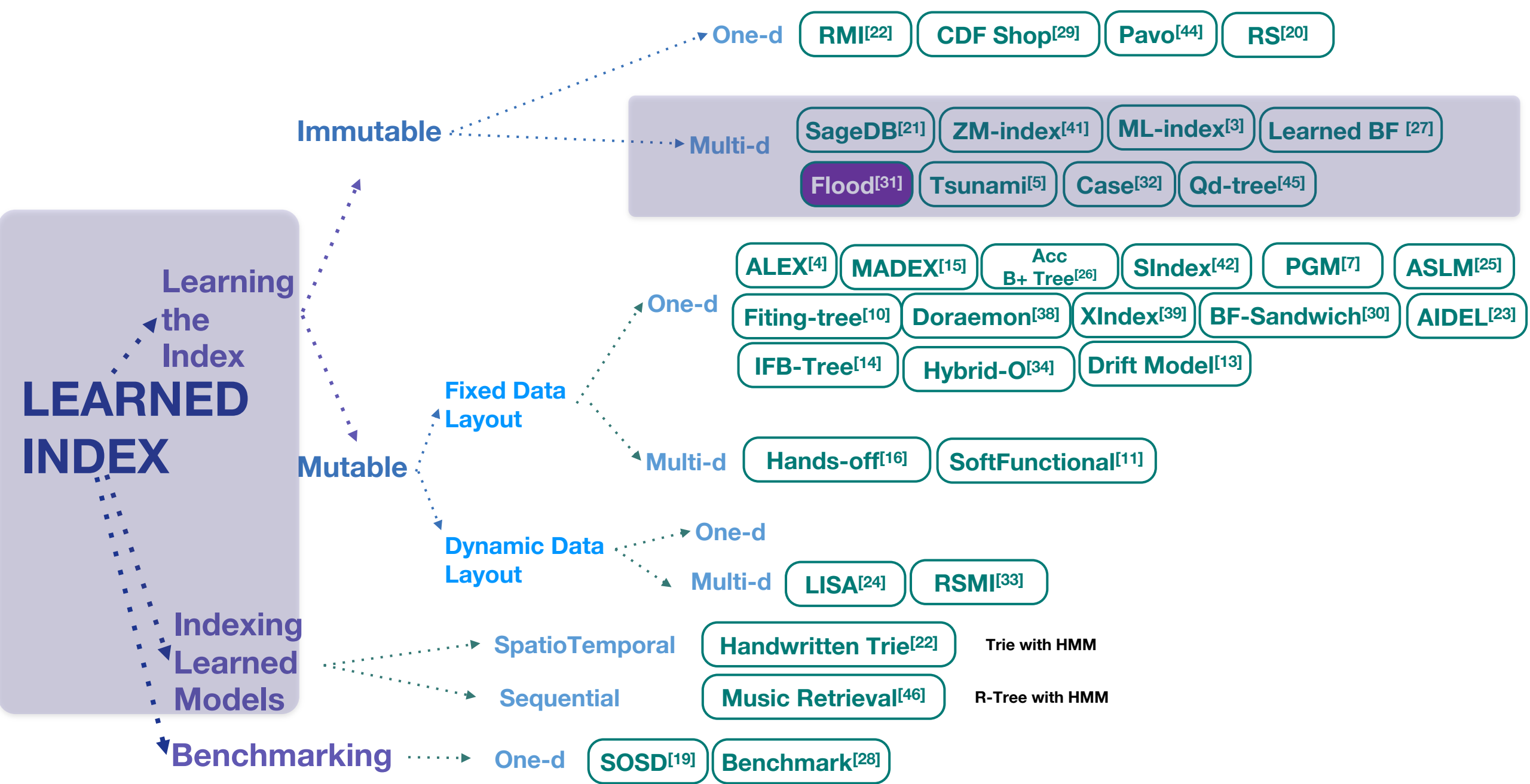
- Minimize the number of blocks/records accessed by a workload
 - Generating block-level layouts with excellent I/O performance
- Query-data routing trees (Qd-trees) are neural network-generated decision trees that
 - Recursively partition the data space into smaller subspaces.
- Use Deep Reinforcement Learning to create Qd-trees
 - Proximal Policy Optimization (PPO)



System Architecture

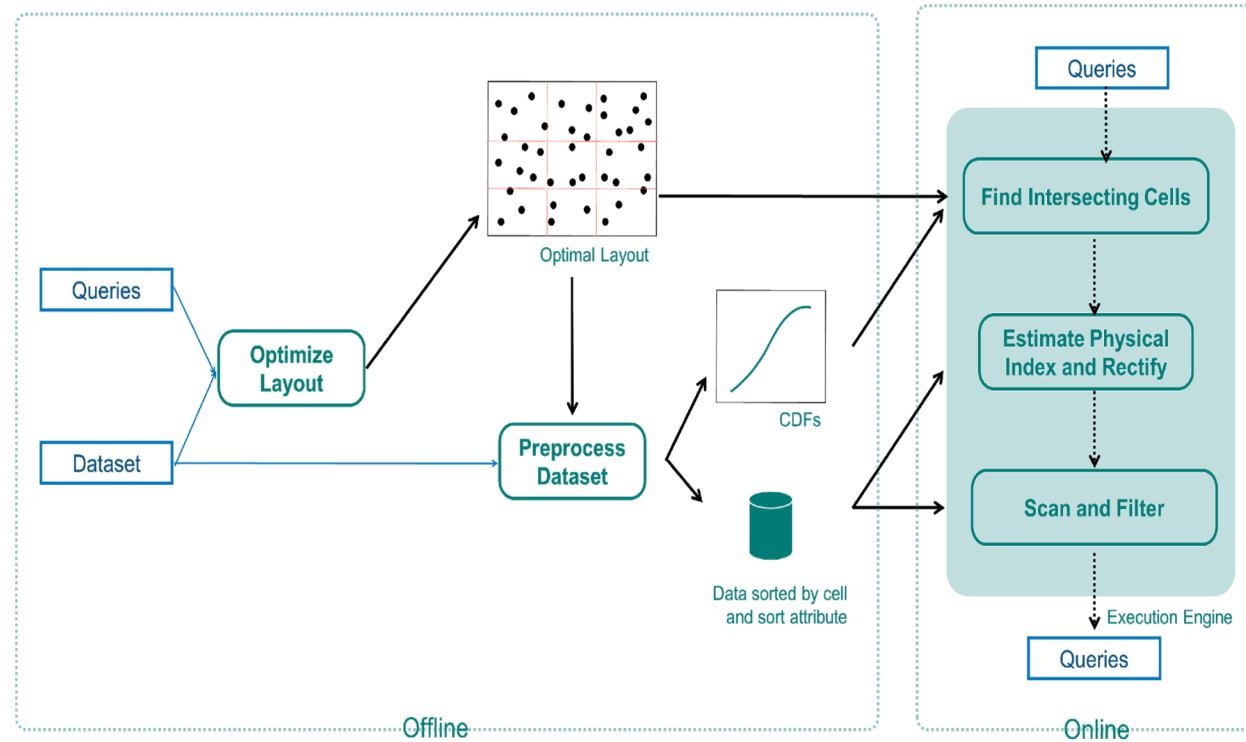
- Experiments over benchmark and real workloads
 - Compared to current blocking schemes:
 - Provides physical speedups of more than an order of magnitude
- For data skipping based on selectivity:
 - Performs within 2X of the lower bound

Taxonomy of Learned Indexes



Core Idea

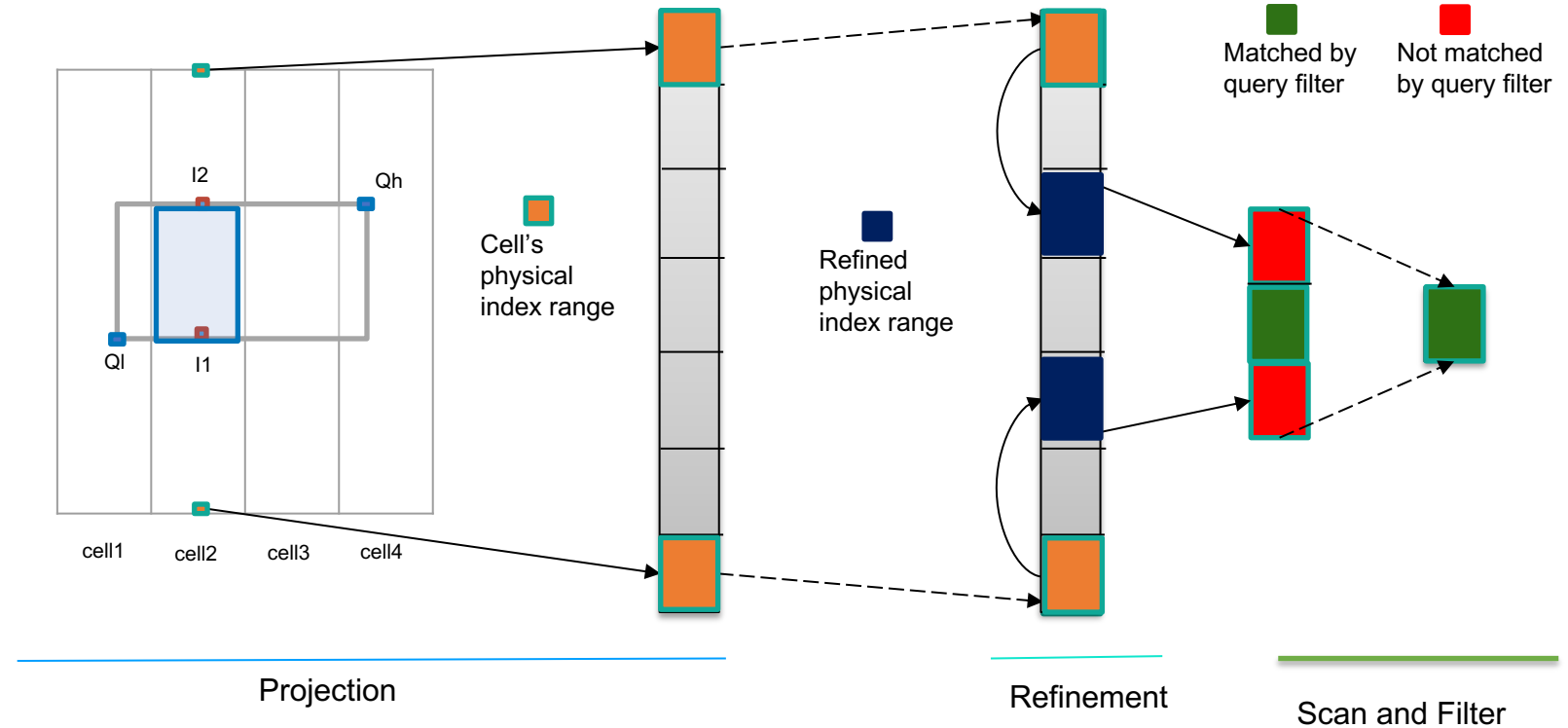
- Proposed index structure: “Flood”
 - Read optimized grid-based index over the multi-dimensional space
 - Co-optimize the data layout and the index structure
 - For particular data and query distributions
- Two components:
 - Offline (pre-processing)
 - Chooses an optimal layout
 - Creates an index based on that layout
 - Online
 - Query execution



System Architecture

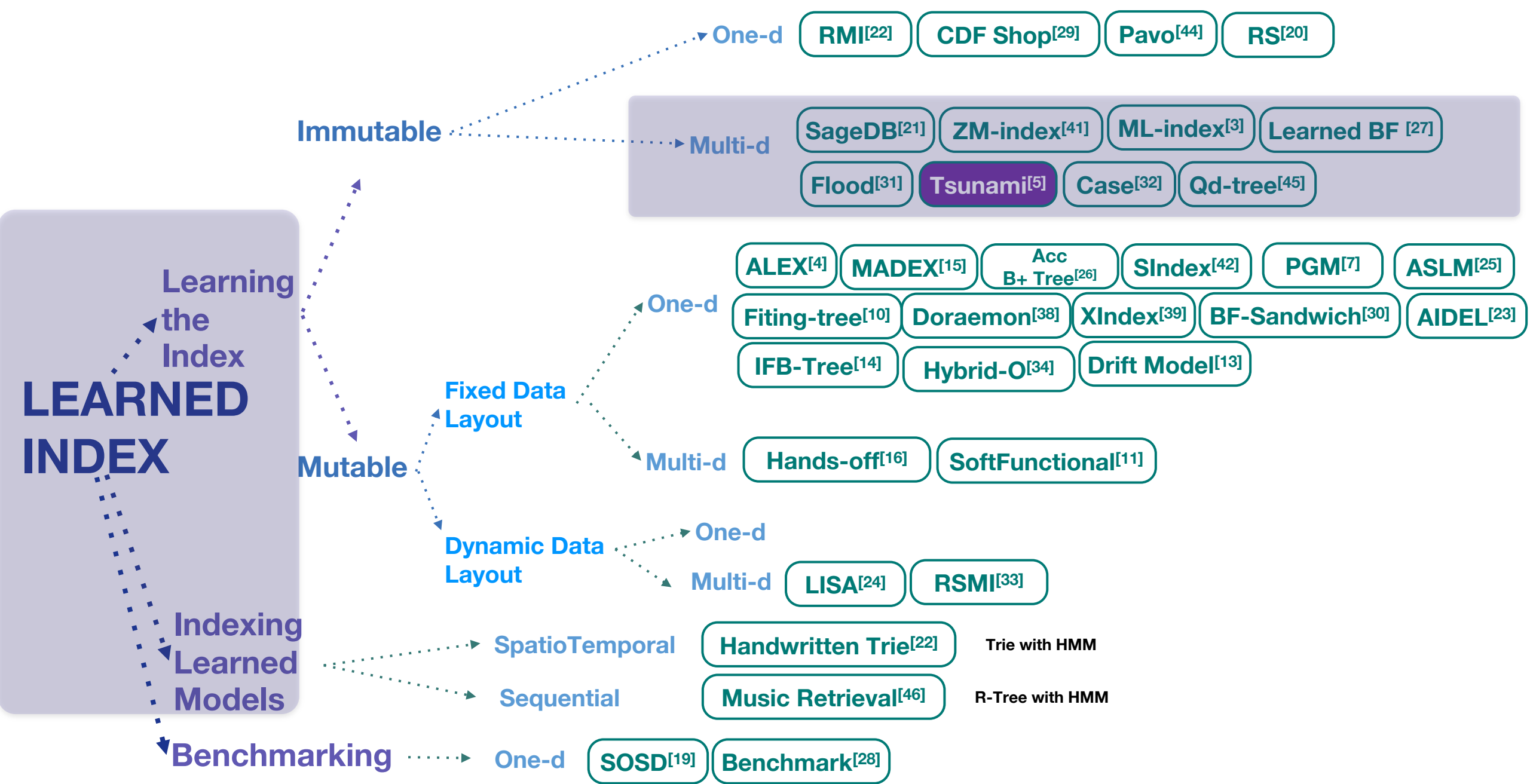
Flood's Workflow

- Projection:
 - Identifies the intersecting cells
 - Identifies the physical index range in each intersecting cell
- Refinement:
 - Utilizes the ordering of points within each cell to refine each physical index range
- Scan and Filter



- Experimental Results:
 - Outperforms optimally tuned spatial indexes (e.g., R*Tree)
 - Uses only a fraction of the space comparing with traditional indexes
- Limitations:
 - Cannot adapt to skewed query workload
 - If dimensions are correlated,
 - Performance and memory usage are affected

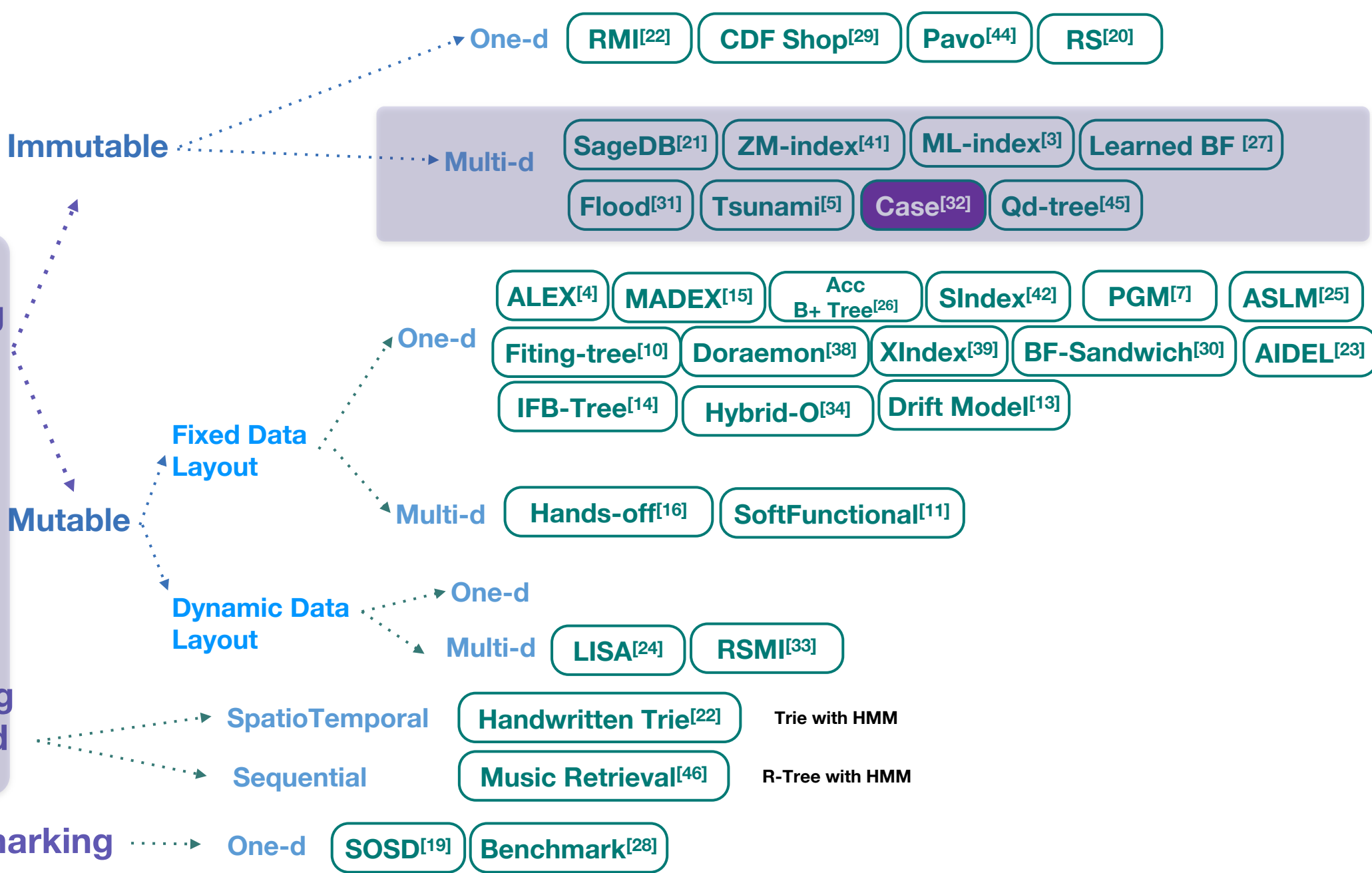
Taxonomy of Learned Indexes



- Extend the idea of Flood to overcome its limitations
 - Adaptable to changes in workload
 - Scales across data size, query selectivity, and dimensionality
 - Up to 6× faster (average query time) than Flood.

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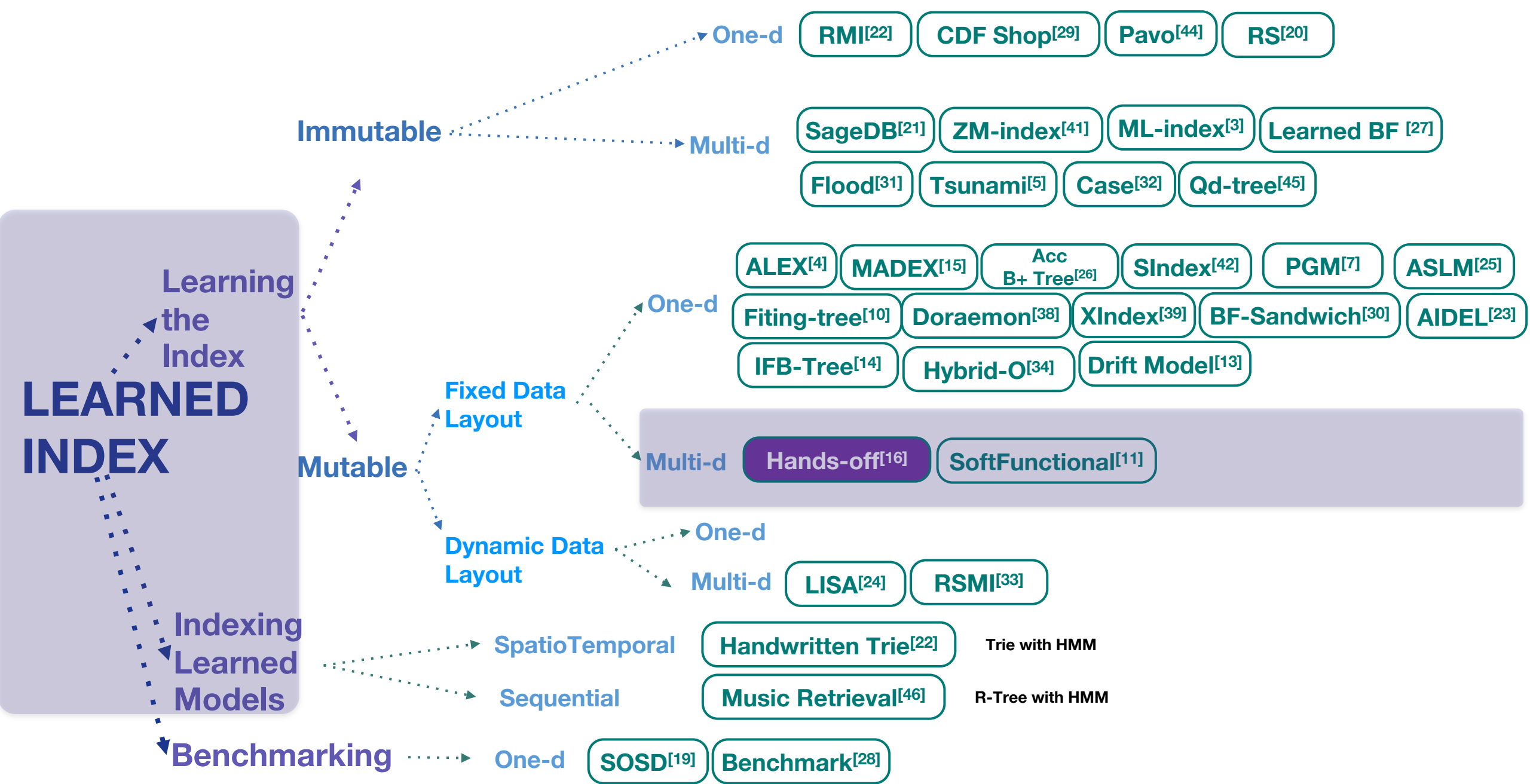
Core Idea

- Apply the techniques in Flood to five other multi-dimensional indexes to answer spatial range queries.
 - Fixed-grid, Adaptive-grid, Kd-tree, Quadtree and STR

Major Insights

- Replace binary search with a learned index within each partition
 - Improve query execution time by 11.79% to 39.51%
- Filter on 1D using traditional index then refine using learned indexes
 - 1.23x to 1.83x times faster than methods that filter on 2D
- Learned indexes are more effective on queries with low selectivity (e.g., 0.00001%) but less effective on queries with high selectivity (e.g., 0.1%).

Taxonomy of Learned Indexes



Motivation

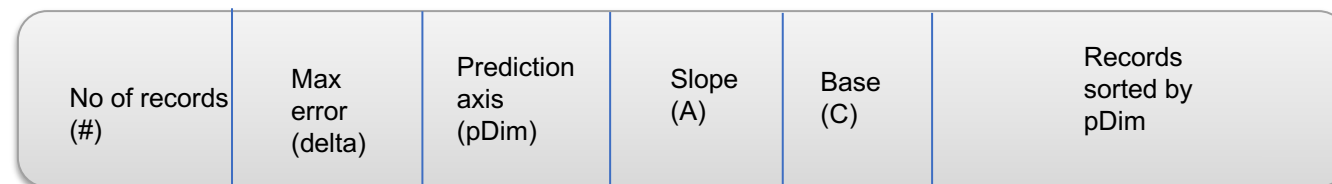
- In-memory hierarchical trees require:
 - Excessive pointer-chasing
 - Time for chasing pointers impacts significantly the query execution time
- New approaches to design indexes are encouraged to utilize the modern hardware platforms
- Updates can be supported by extending any recently proposed update strategies for one-dimensional learned indexes (e.g., ALEX^[4], Drift Model^[13]).

Core Idea

- Interpolation Friendly (IF) Indexes: IF-X
 - X is any multi-dimensional index
- Why Linear Interpolation?
 - Complex models have a higher capacity to fit the CDF
 - But complex models
 - Requires more parameters
 - Slower to compute
 - Linear interpolation is:
 - Simpler
 - Computationally inexpensive
 - Can eliminate expensive training process.

Leaf Node Layout

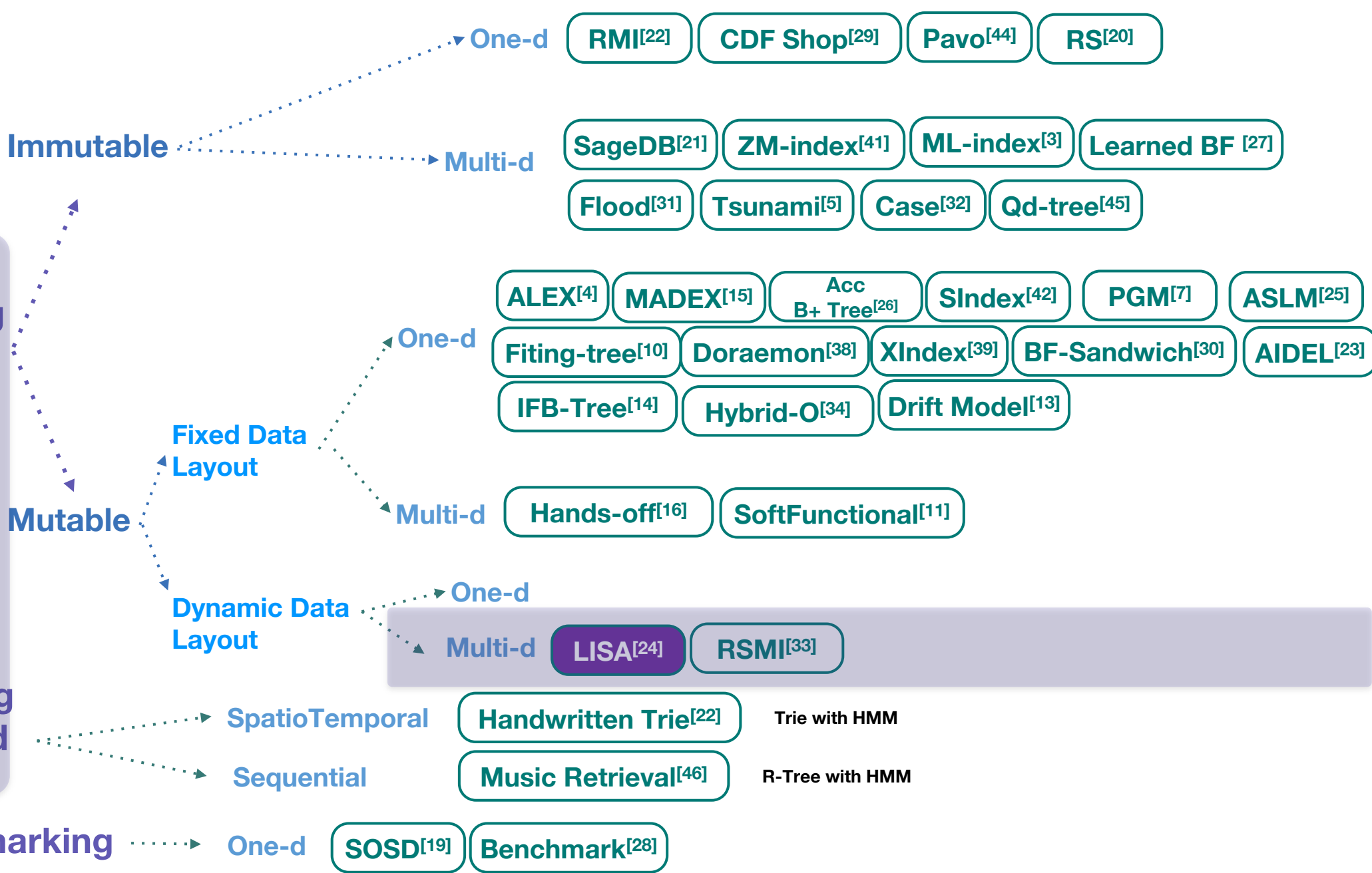
- IF-X indexes sort the records in each leaf node
 - Based on the best order using which the interpolation error is minimized.
- Store all required information in the header of the leaf node
 - No additional computation is needed
- The leaf node structure:
 - pDim: most predictable dimension which is used as the storage order



Performance

- Query execution time can be reduced by up to 60%
- Memory footprint can be reduced by over 90%

Taxonomy of Learned Indexes

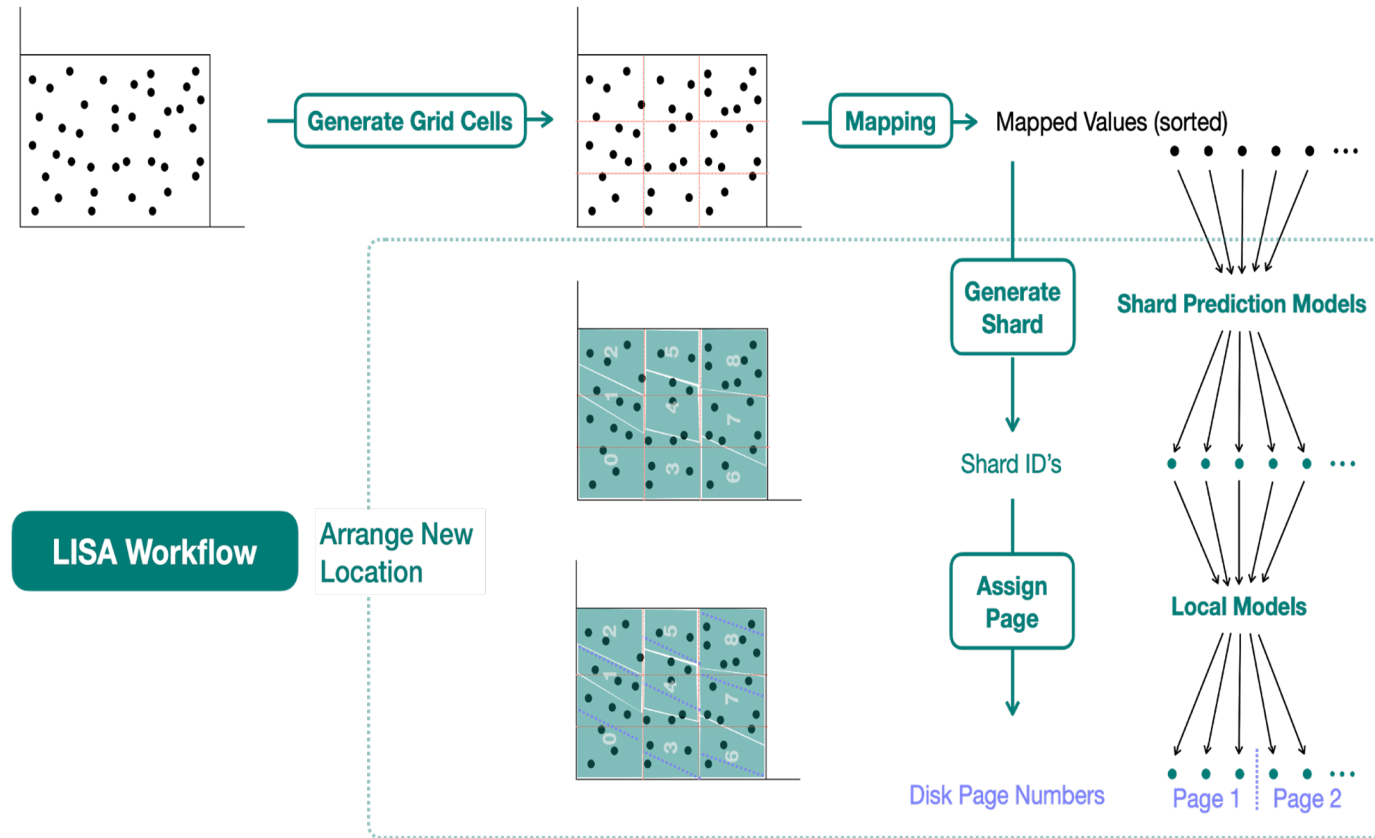


Motivation

- Build a disk-based learned multi-dimensional index for spatial queries.
- Support updates

Core Idea

- Representation of grid cells
- Mapping function:
 - $M(\text{spatial keys}) \Rightarrow 1\text{D mapped values}$
- Learned Shard Prediction Function:
 - $SP(\text{mapped value}) \Rightarrow \text{Shard Id}$
 - Use ML models to generate searchable data layout in disk pages for arbitrary spatial dataset
- Local models:
 - Assign pages for all shards and perform intra-shard operations

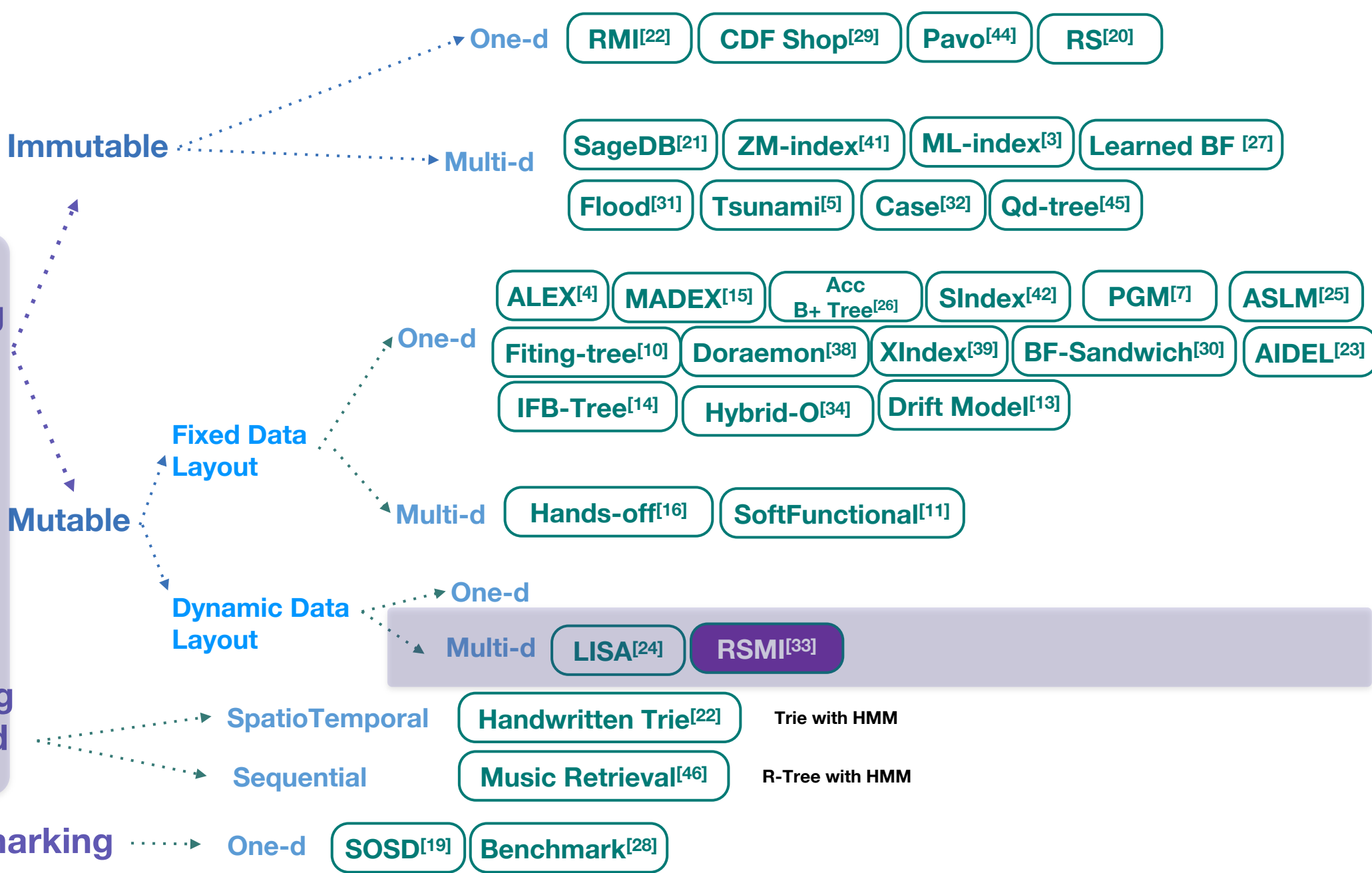


Performance

- Outperforms traditional spatial indexes for range and KNN queries :
 - Memory consumption
 - IO cost

Taxonomy of Learned Indexes

Learning the Index
LEARNED INDEX
Indexing Learned Models

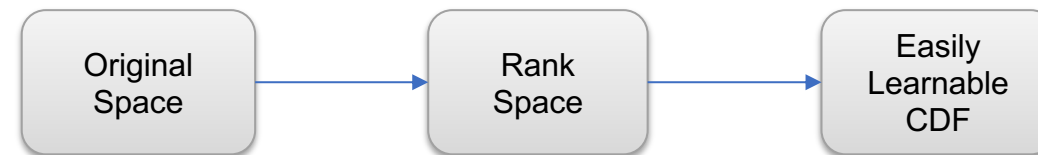


Motivation

- Selecting grid resolution for Z-order for learned multi-dimensional index (e.g. ZM-Index[41]) is difficult:
 - Large cells
 - More false positives due to many points per cell
 - Small cells
 - Hard to learn due to uneven gaps in Cumulative Distribution Function (CDF)

Core Idea

- Spatial index based on ordering the data points by a rank space-based transformation*
 - Simplify the indexing functions to be learned
 - $M(\text{search keys}) \implies \text{disk block Ids (location)}$
- For scaling to large datasets, proposes:
 - Introduce a Recursive Spatial Model Index (RSMI) (in lieu of RMI)
- Support point, window, and kNN queries
- Support updates

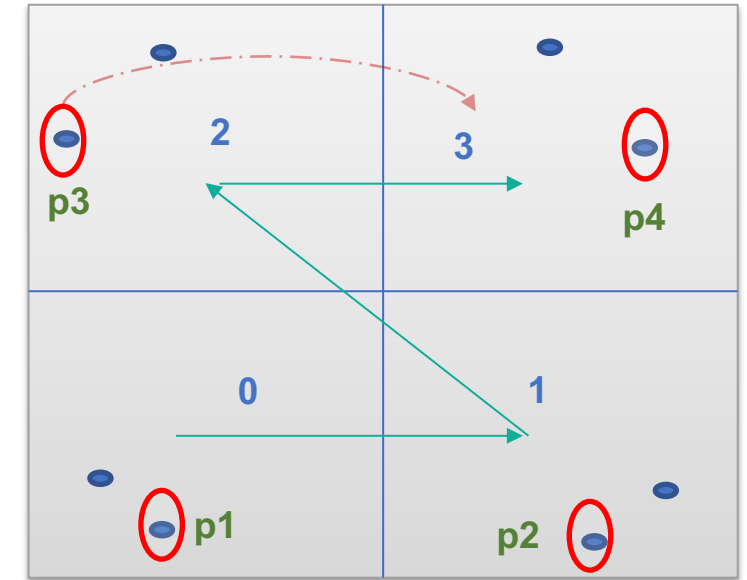


[33] Jianzhong Qi, Guanli Liu, Christian S Jensen, and Lars Kulik. 2020. Effectively learning spatial indices. Proceedings of the VLDB Endowment 13, 12 (2020), 2341–2354.

*[48] J. Qi, Y. Tao, Y. Chang, and R. Zhang. Theoretically optimal and empirically efficient R-trees with strong parallelizability. PVLDB, 11(5):621–634, 2018.

RSMI

- Recursive Spatial Model Index (RSMI):
 - Recursively partitions a dataset
 - Partitioning is learned over the distribution of data
- Steps:
 - Initially distribute the data into equal sized partitions
 - Use a Space Filling Curve (SFC) to assign Ids to partitions
 - Learn the partition Ids using a model $M_{0,0}$
 - Rearrange the data based on the prediction of $M_{0,0}$
 - Recursively repartition
 - Until each partition can be learned with a simple model



Point	p1	p2	p3	p4
Initial partition Id	0	1	2	3
Model predicted Id	0	1	3	3
Learned partition Id	0	1	3	3

Discussion

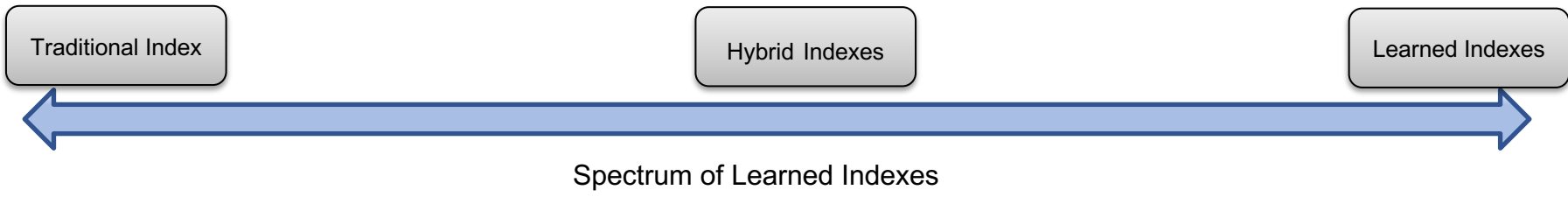
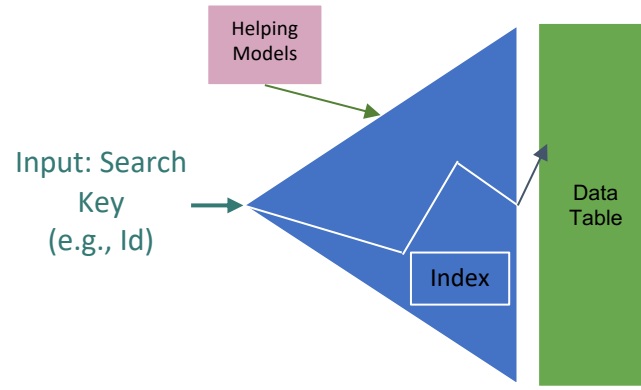
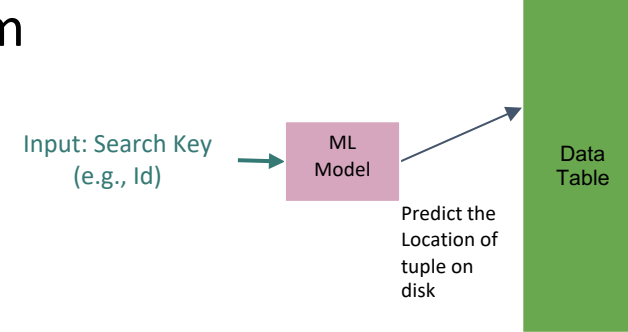
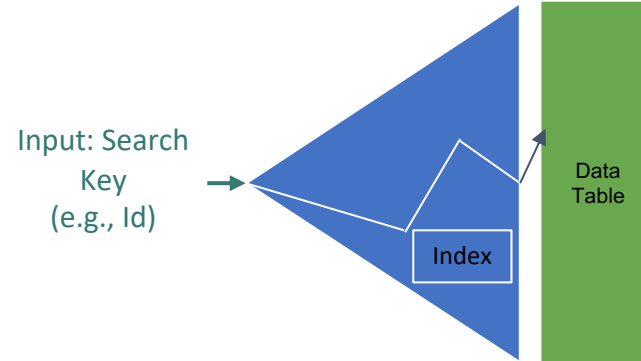
- Window and kNN query results are highly accurate but not exact.
 - i.e., over 87% across a variety of settings
 - Separate mechanism has been proposed for exact answer.
- Does not support query for spatial objects with non-zero extent

Outline of the Tutorial

- Introduction and Taxonomy
- Indexing the Learned Models vs. Learning the Indexes
- Static vs. Dynamic Learned Indexes
- Fixed vs. Dynamic Data Layout
- Learned One-Dimensional Indexes
- Learned Multidimensional Indexes
- **Open Problems and Future Research**

Spectrum of Learned Multi-dimensional Indexes

- *Traditional Indexes:*
 - Theoretical guarantee on performance
 - Well studied and successfully integrated in real systems
- *Learned Indexes:*
 - Learn search-key distribution with some error correction mechanism
 - Better performance with less space requirement
- *Hybrid Indexes:*
 - Optimizing traditional indexes with helping (e.g., ML) models



[14]Ali Hadian and Thomas Heinis. 2019. Interpolation-friendly B-trees: Bridging the gap between algorithmic and learned indexes. In 22nd International Conference on Extending Database Technology (EDBT 2019). <https://doi.org/10.5441/002/edbt.2019.93>

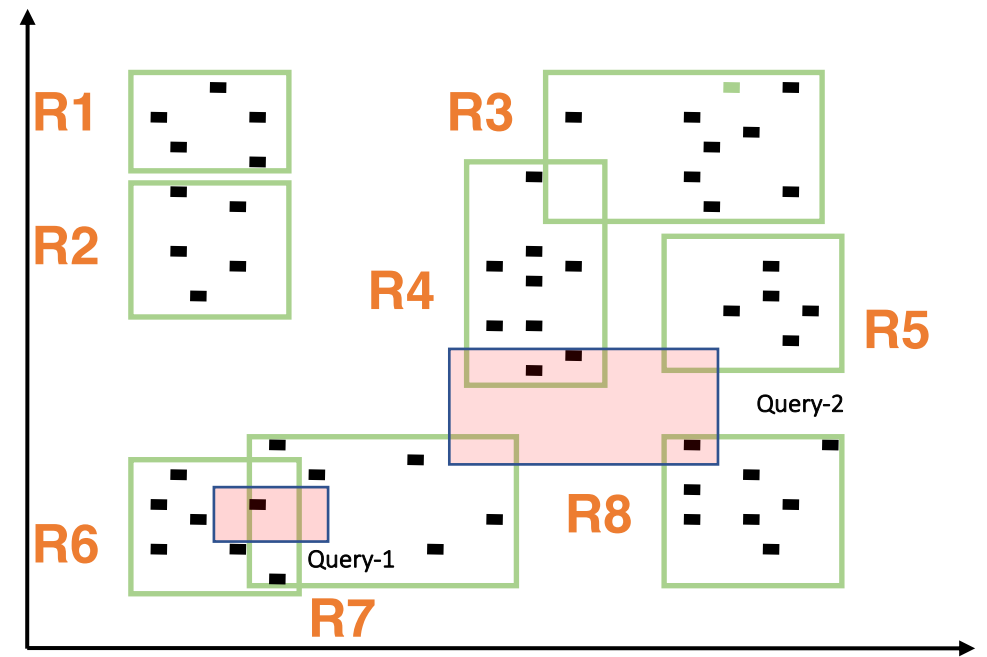
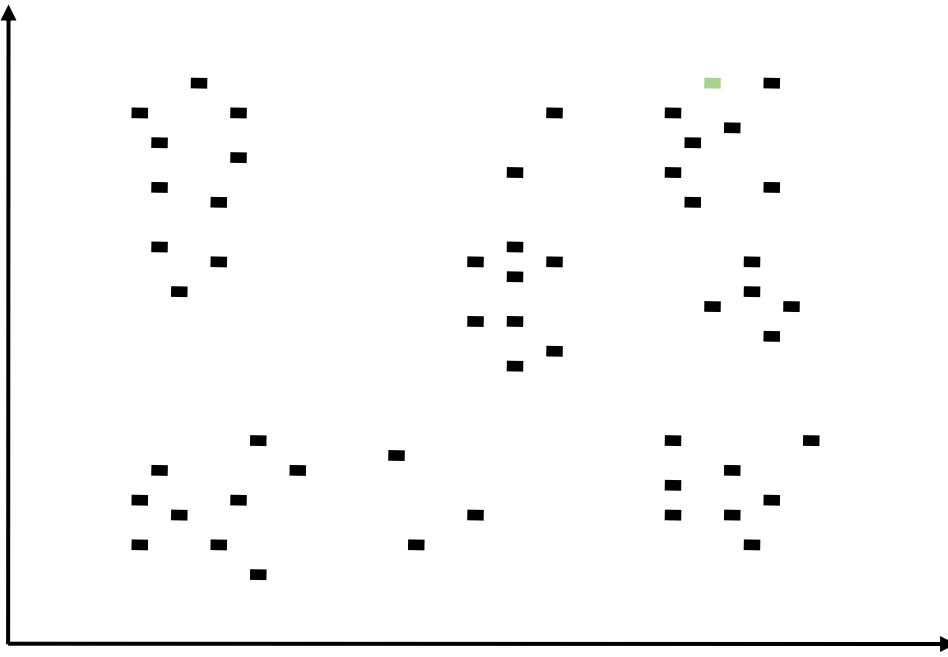
Some Open Problems

- Efficiently support Inserts/Updates
- Support for other spatial operations, e.g., KNN, spatial join, closest pairs
- What types of ML models to use?
- Integrate with real database engines
- Concurrency support
- Develop benchmark for Learned Multidimensional Indexes

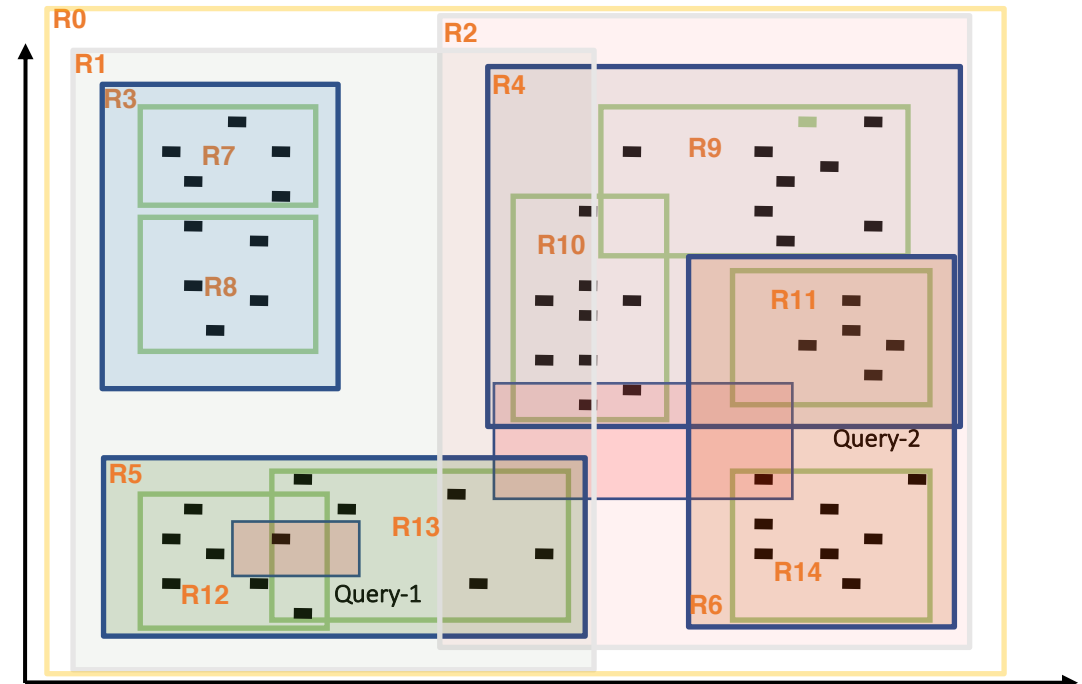
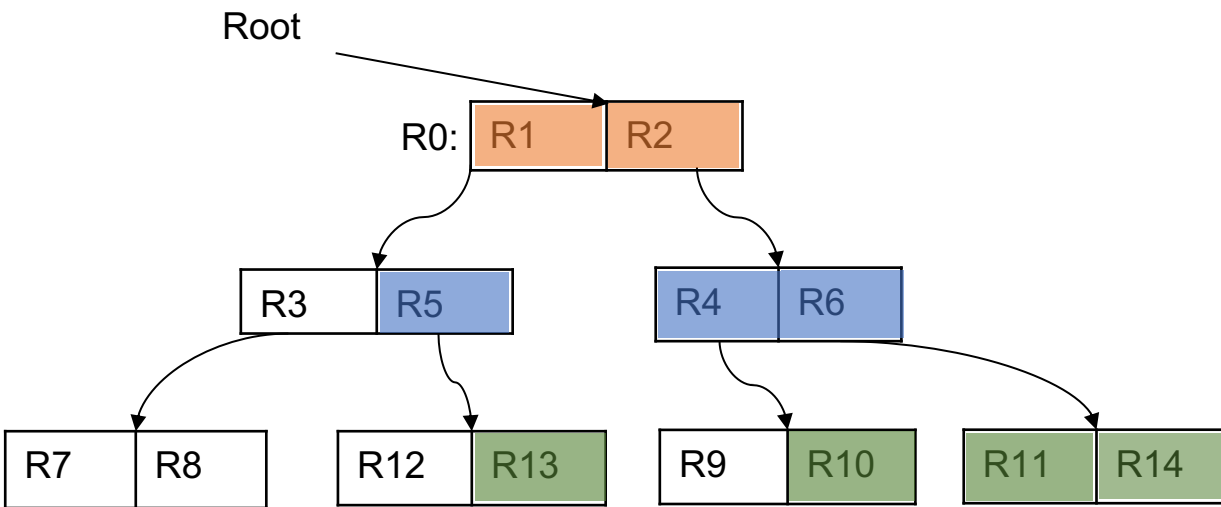
“AI + R”-Tree: Leveraging Machine Learning Techniques in the Context of the R-Tree

Work in Progress

Background – The R-tree

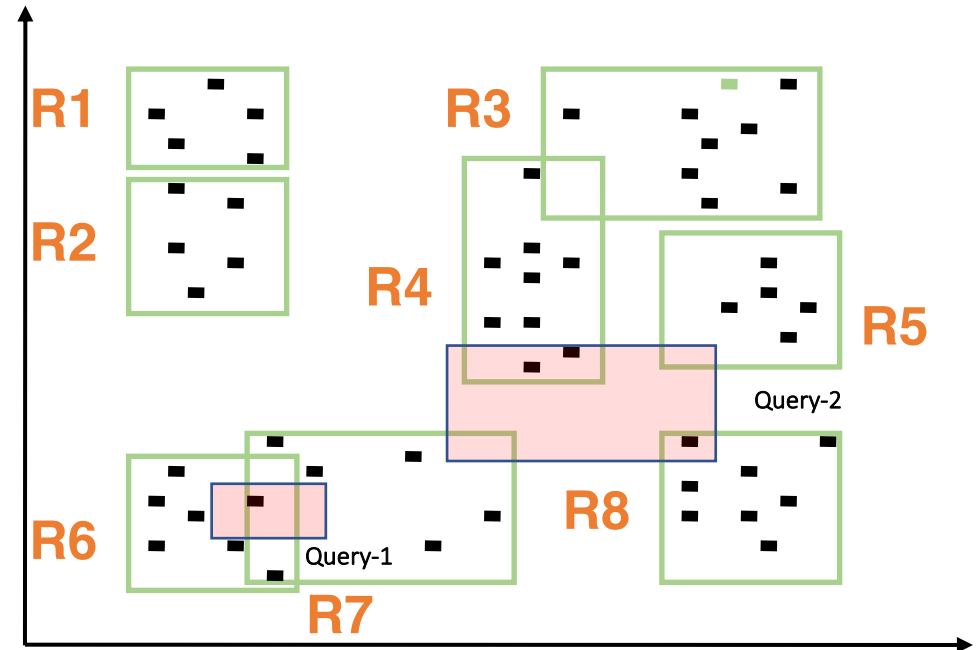


Background – The R-tree (Cont'd)



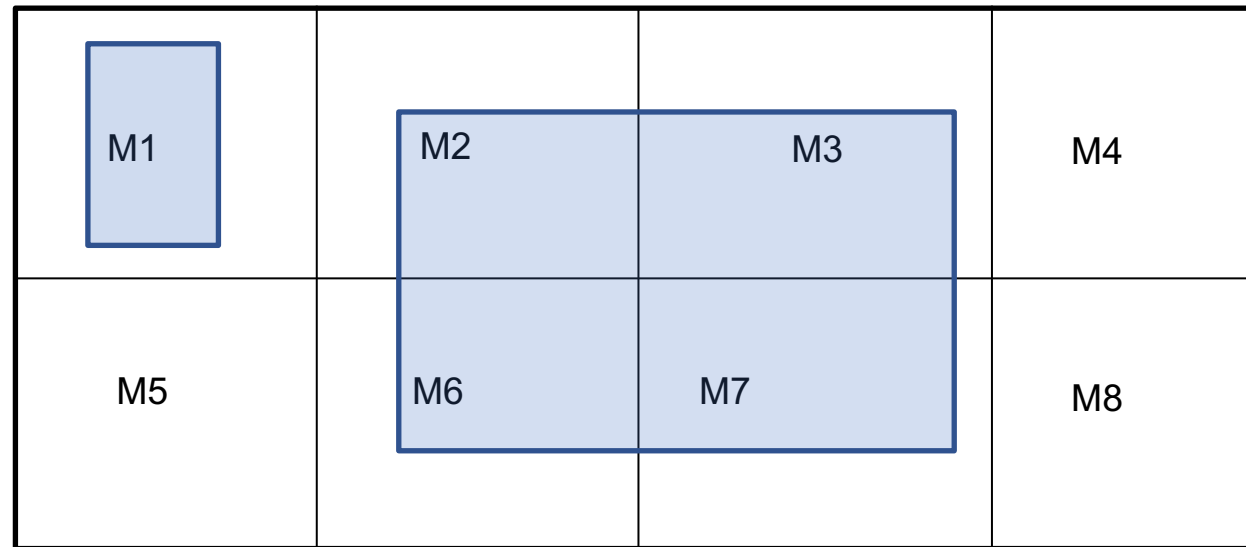
Background – The R-tree (Cont'd)

- For a range query:
 - If the bounding box of a leaf node overlaps the query region, the object itself may not
 - Scan all the data objects contained inside the leaf node
 - Test whether the data object is contained in the given range
 - (or “overlap” the query region for non-zero objects)



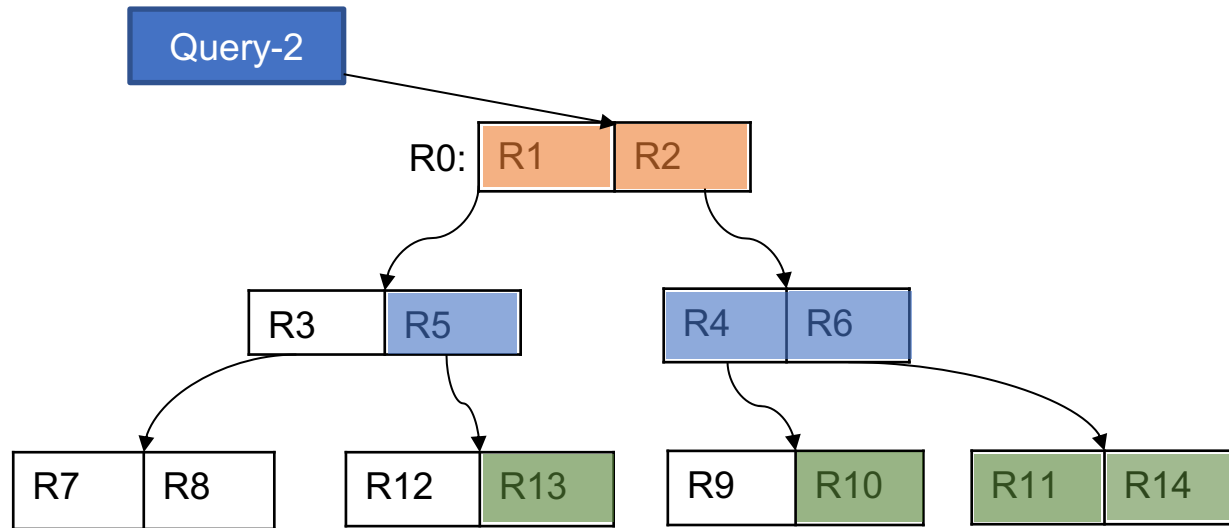
The AI+R tree: A Multi-model Approach

- Using multiple learned models instead of a single learned model
 - M1 is trained only with the queries that fall inside that region.
- Indexing the learned models
- At query time, execute only the corresponding models and produce the aggregated result

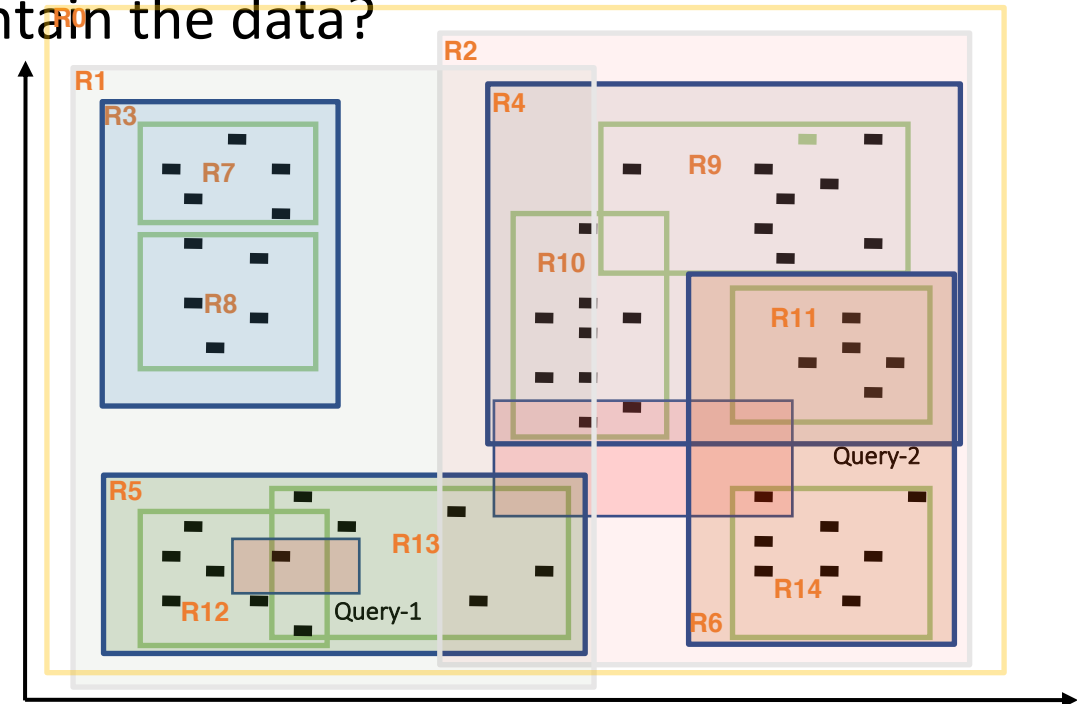


Problem Formulation

- Given a range query $Q(X_{min}, Y_{min}, X_{max}, Y_{max})$ as input, can we directly predict the *leafNodeIds* of a R-Tree that contain the data?



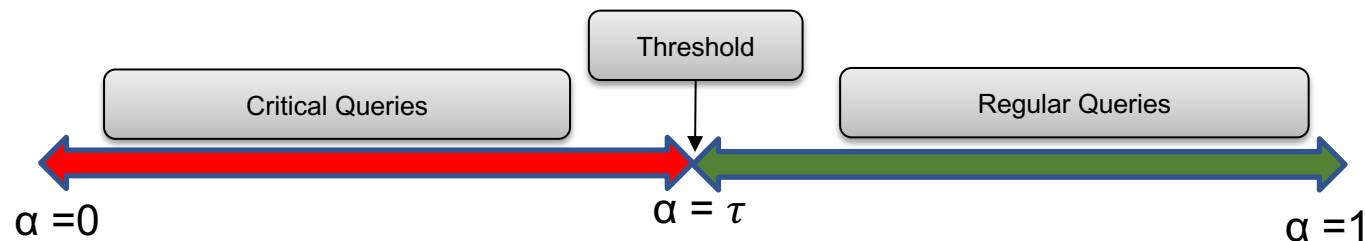
Input	Paths that will be searched by R-Tree	Paths that contains the actual data
Query-1	R1→R5→R12 R1→R5→R13	R1→R5→R13
Query-2	R1→R5→R13 R2→R4→R10 R2→R6→R11 R2→R6→R14	R2→R4→R10 R2→R6→R14



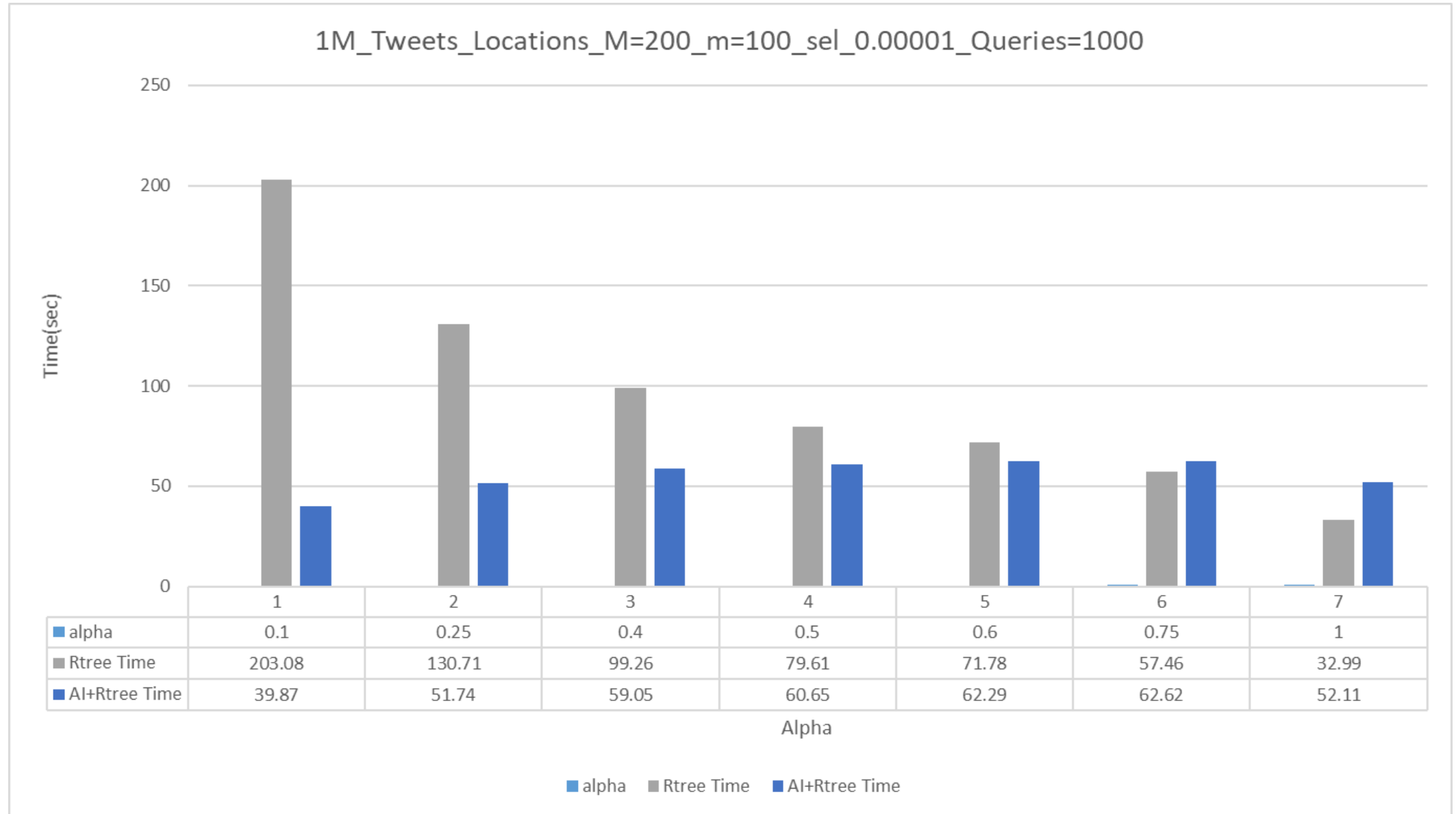
Input	Leaf Node Ids that will be searched by R-Tree	Leaf Node Ids that contains the actual data
Query-1	R12, R13	R13
Query-2	R10, R11, R13, R14	R10, R14

Definition: Alpha

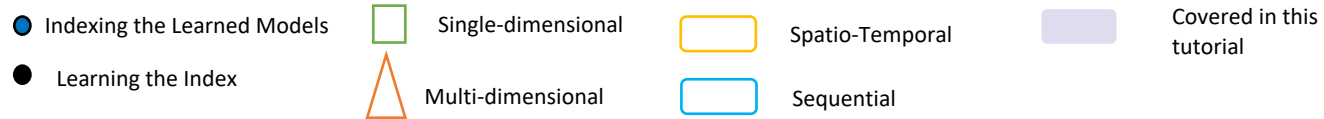
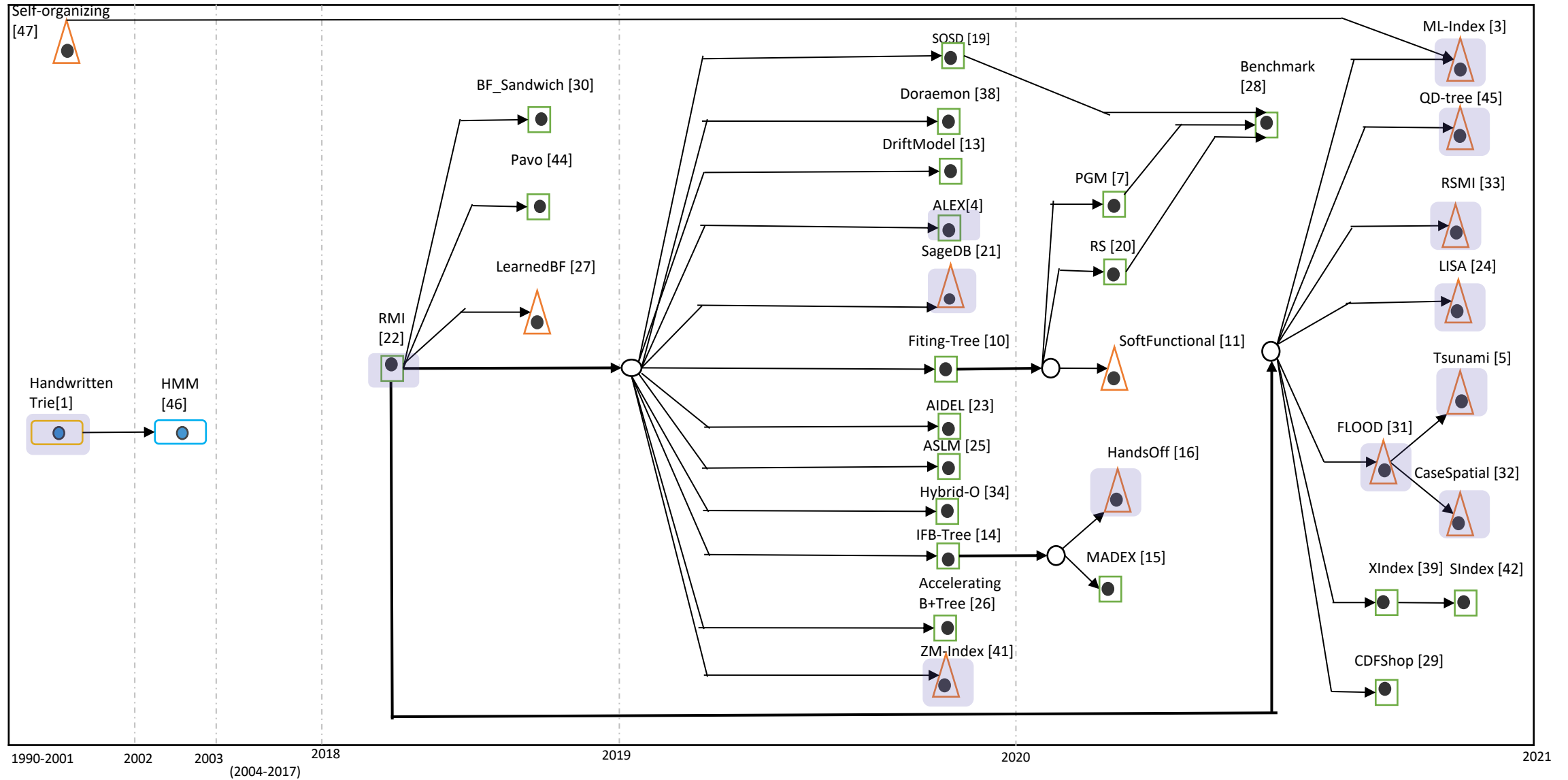
- We define a parameter alpha that reflects the quality of the R-Tree
 - For a range query:
 - $\alpha = \frac{\text{Number of leaf nodes that actually contains the data}}{\text{Number of leaf nodes searched by the R-Tree}}$
 - Reflects the degree of overlap among the nodes.
 - α is in the range $[0,1]$
 - Critical Queries:
 - A range query is declared as a “critical query” if the value of α is less than a pre-defined threshold τ .



1Million_Tweets_Locations



Evolution of Learned Indexes



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Q&A

Website of the Tutorial: <https://www.cs.purdue.edu/homes/aref/learned-indexes-tutorial.html>



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